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THE THEORY and PRACTICE OF THE LONGITUDE AT SEA.

I. By the Reckoning, or uſual Method of SAILING.
II. By the Help of Watches or TIME-KEEPERS.
III. By the SUN, MOON and ZODIACAL STARS.
IV. By New Variation CHARTS of the Magnetic Needle.

WITH

A GENERAL INTRODUCTION to its Aſtronomical and Phyſical Principles; a Variety of Copper Plates and ſeveral uſeful Tables.

By SAMUEL DUNN,
Teacher of Mathematics, LONDON.

LONDON;

Printed for the Author and Sold by him. Alſo, by his Appointment, Sold at ſeveral Bookſellers and Mathematical Inſtrument Makers, in LONDON and WESTMINSTER.

MDCCLXXVIII.

THE THEORY and PRACTICE OF THE LONGITUDE AT SEA.

BY THE AUTHOR.

To

THE HONOURABLE

THE COURT OF DIRECTORS

OF

THE UNITED COMPANY

OF

MERCHANTS OF ENGLAND

TRADING TO

THE EAST INDIES:

THIS WORK,

(DESIGNED AS A COURSE

OF

PLAIN AND EASY INSTRUCTIONS,

FOR FACILITATING

THE THEORY AND PRACTICE

OF

THE LONGITUDE AT SEA)

IS

MOST HUMBLY INSCRIBED,

By

THE AUTHOR,

LONDON,
May 6, 1778.

S. DUNN.

TO

THE HONOURABLE

THE COURT OF DIRECTORS

OF

THE UNITED COMPANY

OF

MERCHANTS OF ENGLAND

TRADING TO

THE EAST INDIES.

THIS WORK

IS DEDICATED AS A TRIBUTE OF RESPECT

FOR THEIR LIBERAL ENCOURAGEMENT

OF THE ART OF NAVIGATION,

BY THEIR MOST OBLIGED

AND MOST HUMBLE SERVANT,

THE AUTHOR.

LONDON,

May 6, 1718.

PREFACE.

THIS Treatiſe is the fourth that hath been publiſhed by me within the Space of four Years laſt paſt, concerning the Diſcovery of the LONGITUDE at SEA, and the Methods whereby that important Problem can be ſolved and applied to the noble Art of Navigation. Theſe Books have not been publiſhed in order one after another as they are to be read, and as they depend on each other; nevertheleſs, they together contain ſuch a continued Courſe of Inſtructions and Operations, as are neceſſary to be known and underſtood in order to a general and particular Knowledge of this moſt intereſting Subject, and it's being practiſed with Succeſs in the moſt important Voyages.

The

The firſt of theſe Books, to be read, is a Treatiſe of *Practical Navigation*, in which are the moſt effectual Methods of determining the Longitude at Sea by the Courſe and Diſtance ſailed, and by help of the obſerved Latitude. But as that Method is found defective;

The ſecond of theſe Books, intitled *A new and general Introduction to Practical Aſtronomy*; contains a Courſe of Inſtructions, for making ſuch aſtronomical Obſervations, by a few inſtruments and ſuch as are eaſily applied; that whoever follows the Plan will ſettle the Longitudes of Places on Land, and particularly near the Coaſts, to all deſirable Perfection, in order to their being inſerted in the Charts.

The third of theſe Books is *this Treatiſe*; in which I have ſhewn how the four moſt uſual Methods of finding the Longitude at Sea may be practiſed with Certainty; alſo, how things which have been taught by others in difficult Rules and Precepts, may be eaſily underſtood and practiſed from original Principles, with many new intereſting Particulars.

The

The fourth Book is a *New Atlas of Variations of the Magnetic Needle*, shewing the Variations from England to the East Indies; which, from many Comparisons that have been made since those Charts were drawn and published, do determine the Longitude at many Places far remote from Land, within a Degree or 60 Miles. It would have been in vain for me to have attempted the drawing of these Variation Charts from many Observations without a Theory, this I have insisted on in the forementioned Works; but by help of that Theory which was discovered by me, and a *few* Observations, the Charts have been drawn, and they agree with actual Observations at Sea, where the Latitudes and Longitudes have been well ascertained.

Having delivered the Theory and Practice of the Longitude at Sea, by the usual Method of sailing in the first of these Works, and the Practice of Astronomy (by the most easy and certain Methods for settling the Longitudes of Places near the Sea Coasts) in the second of these Works; this third Part contains the following particulars which are not in any other

other perſon's Publication. 1ſt. An Examen and Determination of the Errors that may happen to ariſe in the Uſe of Inſtruments and the making of Obſervations both on Land and at Sea. 2d. An eaſy Method of correcting the Errors ariſing by Refraction and Parallax, in determining the Longitude by the Sun, Moon and Stars, the principles of which were invented by me, and inſtanced at the End of my Practical Aſtronomy. 3d. Obſervations on the Lunar Eclipſe of July 30. 1776, to a greater Exactneſs than any Aſtronomer hath thought that Method capable of, and of Uſe for ſettling the true Longitudes of many Places on Land. 4th. An eaſy Method of finding the Requiſites without the Ship's Reckoning; for going through the Computation by the Requiſite Tables. 5th. An eaſy and general Method of finding the Latitude, by having *any* two Altitudes of the Sun, the elapſed Time, and the Latitude ſomewhat near the truth. 6th. The firſt Variation Chart that has ever been drawn from a Theory and found to agree with Obſervations.

A

GENERAL INTRODUCTION,

TO THE

ASTRONOMICAL AND PHYSICAL

PRINCIPLES;

APPLIED IN

THE THEORY AND PRACTICE

OF THE

LONGITUDE AT SEA, &c.

SECTION,

I. *Definitions relative to the Longitude.*

1st. THE Longitude at Sea, is a Spherical Angle, whose angular Point is at one of the Poles of the Earth's Globe; and whose two Sides are Arches of two terrestrial Meridians, one of which passeth through London or

Definitions relative to the Longitude.

ſome other known Place from which Longitude begins to be reckoned, and the other Paſſing through the Place where the Ship is at Sea. And the diſcovery of this Angle, on Board a Ship at Sea, after ſhe hath been a conſiderable time on the Ocean, and her Place is imperfectly known by the Courſes and Diſtances ſailed; is called the diſcovery of the Longitude at Sea.

2d. In this general Definition, it is ſuppoſed that the Earth, on which we dwell is nearly of a globular or ſpherical Form; the Circumference of a great Circle on it's Surface 360 Degrees or 21600 nautical Miles, and its Diameter nearly 6876 nautical Miles, the nautical Mile being to the Engliſh Statute Mile nearly in the Ratio of 8 to 9$\frac{1}{2}$ or of 32 to 37, as it is found by Meaſurement in England and other European Countries. On the Surface of this Globe are the Lands and Seas and theſe are the Baſis or Foundation of Geography and Navigation. But in order to the better underſtanding of this Subject, it is proper to conſider the Motion of the Earth round its own Axis and alſo round the Sun, becauſe from theſe Motions the principal Circles of the Sphere are derived, and on them their Conſtruction chiefly depends.

II. *Of the Motion of the Earth.*

3d. Let *a* repreſent the Centre of the Earth's Globe, and *b c* it's Diameter, *bdce* a great Circle paſſing through the Ends or Extremes of the Diameter, and *dae* another great Circle paſſing through the points *d* and *e* ſuppoſing them equally diſtant from *b* and *c*; then let the whole Globe *bdce* be ſuppoſed to turn round the immoveable Axis *bac* with an equable Swiftneſs, ſo as to perform one whole Revolution in 24 Hours, in a Direction from Weſt toward the Eaſt, and it will repreſent the diurnal or daily Motion of the Earth, whereby the Sun and Stars at great Diſtances from the Earth, appear as though they did move round the Earth's Axis in that ſame time, in a Direction from Eaſt toward the Weſt, although they remain fixed or immoveable.

4th.

Of the Motion of the Earth.

4th. The Points *b* and *c* repreſent the Poles of the Earth. A great Circle as *de* equidiſtant from the two Poles is the Equinoctial, and ſuppoſing an indefinite number of great Circles to paſs through the two poles, and to be very near to each other at the Equinoctial, they are Meridians. From the Equinoctial toward either of the Poles, as *df*, or *gb*, or *gi*, the Latitude of a place is reckoned in Degrees and Minutes, whether it be North or South. But for the Longitude from London, Suppoſing *f* London, and either *g*, *b*, *i*, *k*, or *l*, the Place of a Ship at Sea, the diſtance *dg* in Degrees and Minutes is the meaſure of the Spherical Angle made at the pole by the two Meridians *bd* and *bg*, either Eaſt or Weſt Longitude from London, And the like for Longitude from the Meridian of Greenwhich, Paris, or any other firſt Meridian from which Longitude is reckoned. To theſe Definitions it is proper to add, that, the Zenith is the Point in the Heavens perpendicularly over a Perſon wherever he is. The Nadir is the Point in the Heavens oppoſite thereto directly under him. The Horizon of a Place is either true or apparent. The true Horizon is a great Circle whoſe plane paſſeth through the Earth's Centre, and every part of it's Circumference is 90 Degrees diſtant from the Zenith; but the apparent or viſible Horizon is a great Circle whoſe Plane paſſeth through a Place on the Earth's Surface, and is nearer to the Zenith than the true Horizon by the Semidiameter of the Earth. When a celeſtial Body is at a diſtance from the Earth which is indefinitely great, it makes no Difference which of theſe Horizons is uſed, becauſe they coincide; but for nearer Diſtances a Diſtinction muſt be made, becauſe the Situations of ſuch Bodies are underſtood as referred to the Earth's Centre, whilſt they are viewed from its Surface, and the Difference of theſe poſitions forms the parallax or parallactic Angle.

5th. For the Annual Motion of the Earth, let *a* be the Centre, with the diſtance *ab* of 84000000 Miles draw the Circle *bdce*, and with the diſtance *as* of

Of the Motion of the Earth.

1500000 Miles, let *s* repreſent the common Centre of Gravity of the whole Solar Syſtem, near the Centre of the Sun, which is moving round that Centre, and at the ſame time, the Earth is moving round at that great Diſtance, performing a Revoltion round the Sun in a Year of 365d. 5h. 49m. In this Motion, the Path deſcribed by the Earth is not in the Circle *bdce*, but in an Ellipſis as *bfcg*, in which *b* is the Perihelion or neareſt Approach of the Earth to the Sun; *c* is the Aphelion or fartheſt Receſſion from the Sun, and *f* and *g* are Places in the Orbit where the Earth is at a mean Diſtance from the Sun. This is that great Orb which is deſcribed by the Earth, in a Solar year.

6th. Whilſt the Earth is thus moving round the Sun, let *b* be a point in that Orbit to which the Earth is arrived; and at the mean Diſtance of 241000 Miles, and with an Eccentricity of 17000 Miles, let a ſmaller Elliptic Orbit be drawn repreſenting the Orbit deſcribed by the Moon round the Earth once in a Month, and the Point in this Orbit where the Moon is neareſt to the Earth will be the Perigee, but where fartheſt from the Earth the Apogee, which Terms differ from Perihelion and Aphelion, as one reſpects the Earth and the other the Sun for a Centre.

7th. In this manner the Orbits of the other primary Planets, Mercury, Venus, Mars, Jupiter and Saturn, may be conſidered, with their Perihelion and Aphelion Diſtances from the Sun; alſo the Circumſtances of the Elliptic Orbits of the four Satellites of Jupiter and the five Satelites of Saturn, with the Inclinations of their Orbits to the plane of the Earth's Orbit; but it more immediately relates to the Practice of the Longitude at Sea, to explain the Phænomena ariſing from the Diurnal and Annuel Motions of the Earth, the Motion of the Moon, and the Variety of Appearances ariſing therefrom in the Poſitions of the Sun, Moon and Fixed Stars.

III. *Of the Equation of Time.*

8th. With the Earth's northern Pole elevated above

the

Of the Equation of Time.

the Plane of the Earth's Orbit 66d. 32m. let it be carried in its Orbit from the Aphelion to the Perihelion and with ſuch an increaſing Swiftneſs or Motion, as naturally ariſeth from the Properties of a Central Force; and then with ſuch a decreaſing Motion from the Perihelion to the Aphelion, as ariſeth from the ſame Laws, and the annual Motion will thereby be proved equal, only at two times of the Year, namely, when the Earth is equally diſtant from the Aphelion and Perihelion Points reſpectively. By the Central Force is here meant that Law whereby the Earth moves ſwifteſt in Perihelion and ſloweſt in Aphelion, and therefore is continually under either an accelerated or retarded Motion. And this is manifeſt from Obſervations, of which there are different ones, and if there were no other, ſeeing the Sun's Diameter meaſures 64 Seconds of a Degree more in Perigee than in Apogee, it alone is ſufficient for the Eſtabliſhment of this Truth.

9th. From theſe Principles, by Reaſon of the Earth's Equable Rotation round it's own Axis, the conſtant Parallelism of it's Axis, and the vaſt Diſtance of the Fixed Stars from the Solar Syſtem, the Sidereal Days are equal in Length, that is, there is the ſame Length of Duration from the inſtant a fixed Star appears on the Meridian to it's Return to it, at all times of the Year. But the Solar Day beginning and ending always at Noon, begins at London the Inſtant when the Plane of the Meridian of London produced, paſſeth through the Centre of the Sun; and the like for the Meridians of other places. And therefore, firſt, the ſame Fixed Star appears to come to the Meridian of any Place of Obſervation oh. 3m. 47s. of time nearly, every Day, ſooner than the Sun. Secondly, the Day is different in Length at different times of the year, although by Solar Time Aſtronomers eſtimate their Calculations and Obſervations. Thirdly, the Mean Solar day (ſo called becauſe it is of equal Length from the Beginning to the End of the Solar Year) hath it's Noon ſometimes before and at other times after

the

Of the Equation of Time.

the Sun is on the Meridian. This is the time uſually attempted to be kept by Clocks and Watches; but for reducing the Inſtant of time on any Solar Day to that of mean Solar time, or the contrary, a Table of Reduction ſuited to each day in the year, is formed and commonly called, the Table of Equation of Time.

10th. It belongs to the Theory of Aſtronomy to ſhew how ſuch Equation of time Tables are formed. A well regulated Pendulum Clock, carefully compared with Meridian Tranſits of the Fixed Stars, will ſhew very nearly what the Change of Variation may be at different times of the Year; but this mechanical Trial is what preſuppoſeth the Equable Rotation of the Earth round it's own Axis, beſide the conſtant Parallelism of it's Axis at different times of the Year; alſo an equal Meaſurement of time by the Clock during it's Application, which may not be true. It is therefore more certain to derive the Equation from the Laws of Nature and correct Obſervations. The Inſtant when the Solar Day begins, is, as before noted, when the Plane of the Meridian of a place cuts the Centre of the Sun; conſequently the leaſt Inequality in the Diurnal Rotation, or in the Annual Motion, or in the conſtant Parallel Poſition of the Earth's Axis; either of theſe, muſt, in ſome ſmall meaſure, affect the Equation of time. Two of theſe four things are ſuppoſed conſtant, the Place of the Earth's Centre in her Orbit is ſuppoſed aſſignable from the Central Forces and obſerved Tranſits of the fixed ſtars, but here the Doctrine of thoſe Forces is ſuppoſed perfectly known, and the Aſtronomical Clock is ſuppoſed to keep time without Error.

11th. Theſe are ſome of the Difficulties which are to be overcome, in aſſigning the preciſe Quantity of the Equation of time, and others might be mentioned; but whatever it's Quantity may be on any Day in the Year, if a Clock, Watch or any other Time-keeper be ſuppoſed to keep equal time whilſt it is erring but a ſmall part of a ſecond of time per Day; it depends

Of the Equation of Time.

depends chiefly on the Truth of the Equation Table to correct the Error of the Time Keeper, and on the contrary, if the Equation Table be defective, the Truth or Error of the Time Keeper cannot be known; therefore, in the making of Aſtronomical Obſervations at Sea (where many Impediments may prevent the keeping of equal Time for a long Continuance) it is certainly beſt to have the Solar time either before or after the Obſervation, as ſhortly as poſſible.

12th. As Solar time is moſt commonly uſed, ſo is it moſt eaſily had from actual Obſervations both by Land and Sea. On Land the Beginning of the Solar Day is accurately and eaſily had by obſerving the Inſtant when the Centre of the Sun is on the Meridian of the Place of Obſervation; and although there be no ſuch Convenience at Sea, the time which the Sun is either ſhort of, or paſt the Meridian, on any Day, is eaſily had by Help of a ſmall Proceſs; the Latitude of the Place, the Declination of the Sun, and the Altitude of the Sun being known. In the Sun's Meridian Tranſit, let *a* repreſent the Earth's Centre, *b* the Sun, *cd* the Earth's Axis, *ef* the Equinoctial, *g* a Place on Land or Sea, and *cgedf* the Meridian of the place *g* revolving round the Axis *cd*; and ſuppoſing *bh* and *bi* Tangents to the Meridian, let *g* be ſuch a point therein as approacheth neareſt to *b* in the Rotation; then if *g* coincides with the Plane *bbi*, the Plane of the whole Semicircle *hgi* will alſo coincide with that Plane. And becauſe at the Inſtant this happens, it is Noon along the Meridian from *c* to *e* and thence to *d*, but at the ſame Inſtant Midnight from *c* to *f* and thence to *d*, the ſame Quantity of Equation of Time for that Inſtant muſt be allowed for all Parts of this Meridian reſpectively which are neareſt toward the Sun, for at thoſe places it is Noon, but at the other Parts of the ſame Meridian it is Midnight, when the ſame Equation of Time muſt be allowed.

IV. *Of the Solſtices and Colures.*

13th. Whilſt the Earth is thus moving round the Sun

Of the Solſtices and Colures

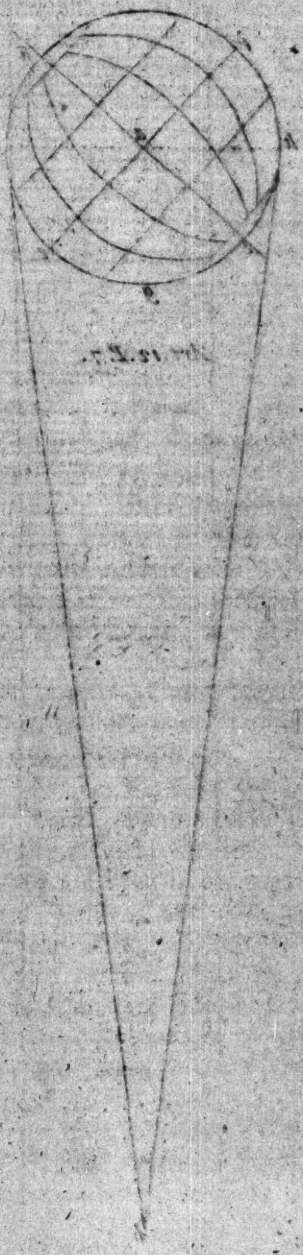
Sun and revolving round it's own Axis, there will be two Points in the Annual Orbit, to which the following Properties will belong whilſt the Earth is ſo revolving. The Plane of a Meridian at Right Angles to the Earth's Orbit, being produced, will cut the Pole of the Equinoctial, and alſo that of the Earth's Orbit; therefore, theſe two Poles are diſtant from each other 23d. 28m. in the Arch of a Meridian in the Heavens, the former apparently immoveable by Reaſon of the conſtant Poſition of the Earth's Axis, and the latter apparently revolving by the Earth's diurnal Rotation. This Meridian is the Solſtitial Colure, and the Points in the Orbit are reciprocally the Beginning of the Signs Cancer and Capricorn, in which the Sun appears as viewed from the Earth, at his greateſt Declination from the Equinoctial; theſe are the limits of the Tropics of Cancer and Capricorn.

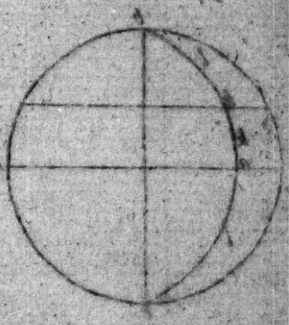
14th. But when the Earth is in either of the two other Points of the Orbit, where her two Poles are equally illuminated by the Sun, he appears in the Plane of the Equinoctial produced, is then ſaid to have no Declination, and the Meridian paſſing through the Sun's Centre and the Pole of the Equinoctial is the Equinoctial Colure, and is that Meridian in which the Sun is, when he is either at the beginning, or middle of the twelve Signs.

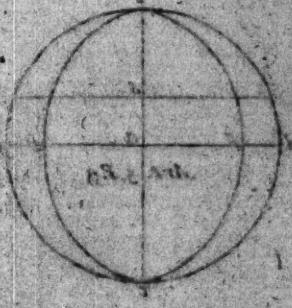
V. *Of Longitude by the Fixed Stars.*

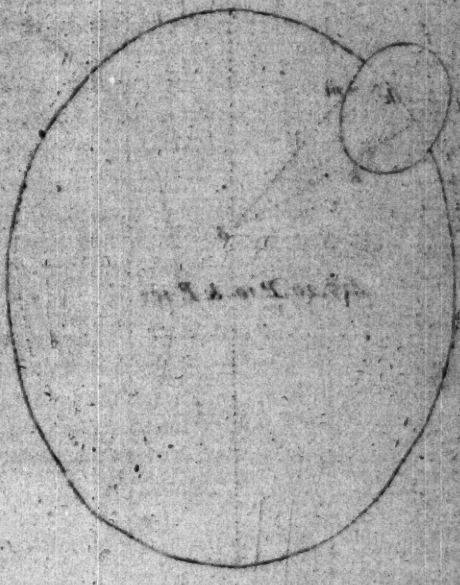
15th. From theſe Cauſes, the Longitude of the Sun, alſo the Declination and the Right Aſcenſion, as they are uſually inſerted in Ephemerides or Aſtronomical Tables, and even the quantity of the Equation of Time itſelf, cannot be true but for the Meridian of ſome certain Place on the Earth, to which theſe Calculations are adapted. And when it is ſaid that the Moon, or any one of the primary Planets, or any one of the Fixed Stars, tranſits the Meridian, at a certain time of the Day for any Place, the time is reckoned from that Meridian. But in the apparent Returns of the Fixed Stars to the Meridian of any Place, they will

be

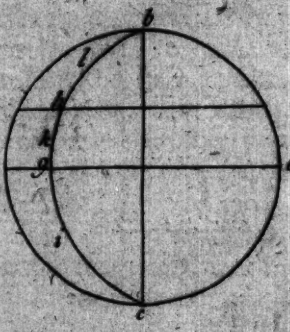


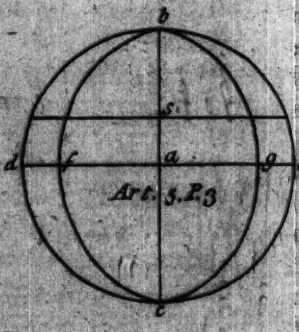




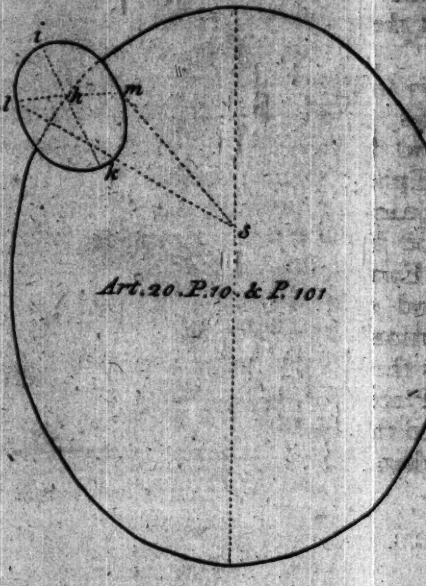


Art. 3. P. 2





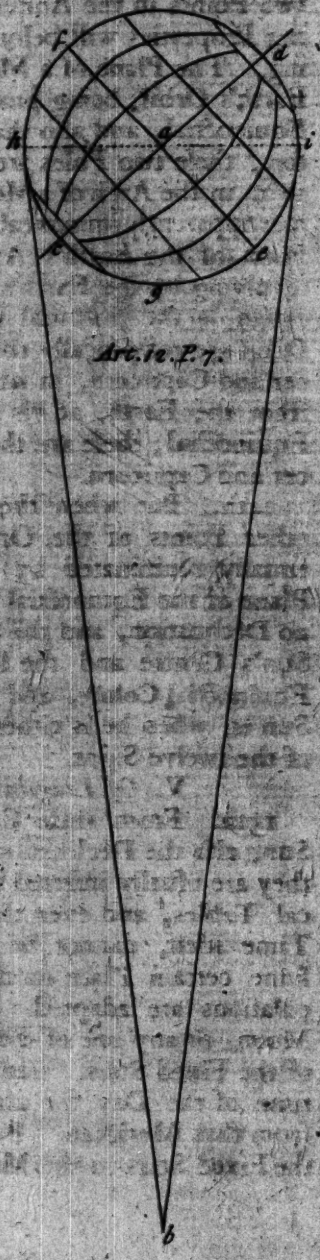
Art. 3. P. 3



Art. 20. P. 10. & P. 101

Published as the Act directs

Plate 1.



Art. 12. P. 7.

st directs July 1. 1778. by S. Dunn.

Of Longitude by the Fixed Stars.

always at a certain determinate Hour, Minute and Second of time, for that place only; consequently an Observation of their Meridian Transits at any other Hour, Minute and Second of time on the same day, shows that the Observer is under another Meridian; and the Difference of their two times, increased in the Ratio of 1 Minutes to 1 Hours, gives the Longitude of one Meridian from the other in time, which is to be turned into Degrees of the Equinoctial.

This is one of the most antient Methods of discovering the Longitude of Places both by Land or Sea, but is embarrassed with such Difficulties, that it never can be brought into Practice and answer to the Exactness that is wanted; because, first, an Error of Seconds of time evidently produceth an Error of 6 Degrees of Latitude by this Method, and an Error of a Second of time in the Observation, an Error of a Degree and half of Longitude by this Method. Secondly, Experience proves that neither correspondent Transits, nor single ones, can be easily taken to that Accuracy, whereas the Longitude of a Place is wanted to be known at least ten times as Exact for Land and at Sea. this Method is wholly impracticable. So that all Reliance for the Longitude, both by Land and at Sea, are impracticable by the apparent Separation of the Sun from the Fixed Stars. Nevertheless, this Method is applicable for the Discovery of the Longitude by the apparent Separation of the Primary Planets from the Sun, but particularly of the Moon from the Sun and Fixed Stars.

VI. Of the Eclipse and Zodiac.

In the Syftem of the world the Sun being supposed to be at Rest near the Centre, on Account of the Motion of the Orbs he describes and the vast Distance of the Planets, the Planet Mercury at the Distance of 37 Millions of Miles from the Sun moves round him in 87 Days. The Planet Venus at 69 Millions of Miles. Like Earth at [illegible] The Planet Mars [illegible]

Of Longitude by the Fixed Stars.

be always at a certain determinate Hour, Minute and Second of time, for that place only; conſequently an Obſervation of their Meridian Tranſits at any other Hour, Minute and Second of time on the ſame day, ſhews that the Obſerver is under another Meridian; and the Difference of theſe two times, increaſed in the Ratio of 4 Minutes to 24 Hours, gives the Longitude of one Meridian from the other in time, which is to be turned into Degrees of the Equinoctial.

16th. This is one of the moſt antient Methods of diſcovering the Longitudes of Places both by Land or Sea, but it is embaraſſed with ſuch Difficulties, that it never can be brought into Practice and anſwer to the Exactneſs that is wanted; becauſe, firſt, an Error of 4 Seconds of time evidently produceth an Error of 6 Degrees of Longitude by this Method; and an Error of a Second of time in the Obſervation, an Error of a Degree and half of Longitude by this Method. Secondly, Experience proves that neither correſpondent Tranſits, nor ſingle ones, can be eaſily taken to that Accuracy, whereas the Longitude of a Place is wanted to be known, at leaſt ten times as Exact for Land; and at Sea, this Method is wholly impracticable. So that all Reſearches for the Longitude, both by Land and Sea, are impracticable by the apparent Separation of the Sun from the Fixed Stars. Nevertheleſs, this Method is applicable for the Diſcovery of the Longitude by the apparent Separation of the Primary Planets from the Sun, but particularly of the Moon from the Sun and Fixed Stars.

VI *Of the Ecliptic and Zodiac.*

17th. In the Syſtem of the world (the Sun being ſuppoſed nearly at Reſt near the Centre, on Account of the Smallneſs of the Orbit he deſcribes and the vaſt Diſtance of the Planets) the Planet Mercury at the Diſtance of 32480000, Miles from the Sun moves round him in 87d. 23h. 15m. The Planet Venus at 60677000 Miles in 224d. 16h. 50m. The Earth at 83970000 Miles in 365d. 5h. 40m. The Planet Mars at

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127841000 Miles in 686d. 23h. 27m. The Planet Jupiter at 436388000 Miles in 4332d. 12h. The Planet Saturn at 800271000 Miles in 10759d. 7h. And one entire Revolution of the Moon round the Earth, by a Medium, is performed at the Diſtance of 238000 Miles in 27d. 7h. 46m.

18th. The Planes of all the Orbits of theſe Planets, are partly above and partly below the Plane of the Earth's Orbit. The Inclination of Mercury's Orbit to that of the Earth is 6 degrees 55 minutes. Venus's 3d. 25m. Mars's 1d. 52m. Jupiter's 1d. 20m. Saturn's 2d. 30m. And the Mean Inclination of the Moon's Orbit 5d. 18m. Therefore, all the Primary Planets, and even the Moon, by Reaſon of the ſmallneſs of thoſe Angles, appear to trace out a Path in the Heavens amongſt the Fixed Stars, not many Degrees diſtant from the Path of the Sun, in making their compleat Revolutions.

19th. The Path of the Sun apparently deſcribed amongſt the Fixed Stars, is the Ecliptic, and the few Degrees of Breadth North and South thereof in which the Planets appear, conſtitute the Zodiac; and the Fixed Stars which are ſituated within the Northern and Southern Bounds of the Zodiac, are now known by the Name of Zodiacal Stars. But the largeſt of thoſe Stars only and ſome others that are without the Zodiac, adjacent to it, are moſt commonly applied in Obſervations for the Longitude at Sea. For theſe Reaſons (and particularly on Account of the Smallneſs of the Angles which the Orbits of the Planets make with the Plane of the Earth's Orbit) the Planets always appear in ſome Part of the Zodiac, and to ſeparate in Diſtance from the Zodiacal Stars; and particularly the Moon, whoſe apparent Separation is greateſt on account of her nearneſs to and quick Revolution round the Earth, the mean Motion in her Orbit being 13d. 10m. per Day.

VII. *Of the Phaſes of the Moon.*

20th. From the ſame Cauſe ariſeth the Appear-

Of the Phaſes of the Moon.

ance of the Face of the illuminated Part of the Moon, from the Change to the Full and from the Full to the Change. One part of the Orbit deſcribed by the Moon round the Earth, would be within the Earth's Orbit toward the Sun, and the other without toward the Fixed Stars, if the Earth were at Reſt; and therefore, the Moon being a round Body and receiving her Light from the Sun; at her neareſt approach to the Sun, ſhe diſappears, this cauſeth the Change or New Moon; but when moſt remote from the Sun, the Full Moon. Conſequently, from the Change to the Full, and from the Full to the Change, there is always leſs than 180 Degrees of Diſtance from the Moon's illuminated Edge or Limb to the Sun; from the Change to the Full, that Diſtance is increaſing Eaſtward, but from the Full to the Change, that Diſtance is decreaſing Eaſtward; and hence it is that the Moon is ſaid to be Eaſt or Weſt from the Sun, on any given Day in the Year, and the like is underſtood of any of the Zodiacal Stars being Eaſt or Weſt from the Moon.

21ſt. Whatever Latitude a Perſon happens to be in either by Land or Sea, the Elevation of the Pole is always equal to the Latitude of the Place in Degrees and Minutes, this is a Conſequence of the Earth's Axis being in the ſame Poſition as the Axis of the Equator in the Heavens. On this Account, the Planes of all the Parallels of Latitude on the Earth muſt neceſſarily cut the Horizon under one and the ſame Angle, and in like manner muſt all the Planes of the Parallels of Declination in the Heavens. Therefore, the Fixed Stars apparently move through their own reſpective Arches in the Heavens from Riſing to Setting, as obſerved in any Latitude, and the like for the Sun, Moon and other Planets, except what ſmall Differences ariſe by the Change of their Declinations. But the apparent Ecliptic in the Heavens (formed as before noted by the Annual Motion of the Earth) is always one half above and the other half below the Horizon, in any Latitude; and its Poſition not only to the Horizon

Of the Phaſes of the Moon.

but to every part of the viſible Hemiſphere is continually in a State of Change and Alteration. This is the Reaſon why it is difficult to determine the Poſition of the Ecliptic at all times in the Night Time, and without this it is not eaſy to know the Zodiacal Stars.

22d. To this Inconvenience, Nature in the Conſtruction of the Solar Syſtem hath provided an ample Remedy. A Right Line paſſing through the Centres of the Sun and Moon, will paſs through the Middle of the illuminated Part of the Moon's Surface, and the two Corners or Cuſps of the Moon's enlightned Part, will appear more remote from the Sun than any other part of the Moon; therefore, a Line drawn at Right Angles to the Line joining the Cuſps, both to the Eaſt and Weſt, will ſhew the Direction in which the Ecliptic is at all times and places; and in the Night Time, in this Line or near it, the Obſerver ſhould look for the Zodiacal Stars. The like Method is to be uſed for finding the Primary Planets, making ſuch allowance as is neceſſary for their Latitudes or Northing and Southing from the Ecliptic. And although the largeſt or brighteſt Stars in the Heavens do not happen to be near this Ecliptic, there are ſeveral which are of the firſt and ſecond Magnitude, but a little Diſtance therefrom, which are eaſily diſtinguiſhed by their ſituations to one another and to ſmaller Stars near them; theſe are the principal Zodiacal Stars uſed in finding the Longitude at Sea by the Moon, and their Names are annexed to them in that Chart.

VIII. *Of Longitude by the Moon.*

23d. Seeing then that the Time is ſo ſhort in which the Moon makes an whole Revolution round the Earth, and thereby appears to make a Revolution through the Heavens and near to the Ecliptic, ſhe appears to ſeparate from the Fixed Stars 13 or 14 Degrees in a Day, and from the Sun 12 or 13 Degrees in a Day, which is near 13 times as ſwift as the Separation of the Fixed Stars from the Sun. Conſequently, if the Moon's Place in Longitude can be accurately predicted,

Of Longitude by the Moon.

predicted, in Solar Time, for any known Meridian, the Advantages of deriving the Longitude by Land or Sea (from Obſervations relative to the Angular Diſtances of the Sun and Moon or the Moon and Stars) will be thirteen times as correct as the Method by the Separation of the Sun and Stars only. This is the Baſis or Foundation, on which the Method of finding the Longitude at Sea depends; and is alſo preferable to the Application of the Primary Planets, on Account of the Slowneſs of their apparent Separations from the Sun and the other Celeſtial Bodies. If this Separation was exactly 12 Degrees per Day, it would be 2 Minutes of a Degree in a Minute of Time; and therefore, in ſuch Caſe, an Error of 2 Minutes of a Degree in Longitude, either in the Prediction of the Moon's Place, or Obſervation for that Place amongſt the Zodiacal Stars, or in both, would produce an Error of one Degree in Longitude on the Earth's Globe; which, at the Equinoctial is 60 Nautical Miles.

24th. To the Care of Aſtronomers who are provided with the moſt perfect Inſtruments that can be made, it is left to make the beſt Obſervations they are able to make, in order that they may be compared with the beſt Theories of the Celeſtial Motions, that can be Invented, and that thereby the Predictions concerning the Places of the Sun, Moon, Planets and Fixed Stars, may be inſerted in Tables, for Time to come, and be found to agree with actual Obſervations. This is the Chief End and Deſign of Practical Aſtronomy, the Buſineſs for which the moſt Curious Obſervatories have been erected and furniſhed with Inſtruments in various Places throughout the World, many of which have been with a Deſign to obtain an Accurate Knowledge of the Motions of the Celeſtial Bodies, and make them ſubſervient to the Improvement and Perfection of Geography and Navigation.

25th. In making theſe Obſervations in eſtabliſhed Obſervatories, the Principal Inſtruments uſed, are the Aſtronomical Clock beating Seconds of Sideral Time; the Tranſit Inſtrument placed in the Plane of the Meridian, and having five Wires in it Parallel to each other,

which

Of Longitude by the Moon.

which the Obſerver takes the apparent Tranſit of the Sun, Moon or Star by, according to the Time ſhewn by the Clock; and the Meridian Quadrant, for taking the Meridian Altitude of the Celeſtial Body, or either its upper or lower Limb above the Horizon of the Place of Obſervation. Of theſe Particulars, the Reader may be more fully informed by reading the Beginning of my Treatiſe on *Practical Aſtronomy*.

26th. The Earth's Rotation round it's own Axis being performed in 24 Hours, it follows that a Place at the Equinoctial rolls at the Rate of a Degree in 4 Minutes of Time and a Nautical Mile in 4 Seconds of Time; on which Account, an Error of 4 Seconds of Time any how introduced, would produce an Error of a Nautical Mile at the Equinoctial, if it depended on the Diurnal Rotation only. But the Moon's Separation being from 12 to 14 Degrees per Day; it will be by Inverſe Proportion, as 360d. to 1 Mile, ſo 12d. to 30 Miles; or ſo 13d. to near 28 Miles; or ſo 14d. to near 26 Miles. And therefore, an Error of 4 Seconds of time, or what amounts to the ſame thing, an Error of a Minute of Equatorial Motion or Diſtance in the Heavens, this in obſerving the Moon's Place, produced an Error of 25 Nautical Miles or more at the Equinoctial. It might have been ſaid 30 Engliſh Miles.

IX. *Of the Accuracy of Obſervations.*

27th. Aſtronomical Obſervers being ſuppoſed to have their mental Faculties equal to each other, may be (with Reſpect to their Ability for viewing the Celeſtial Bodies) of three Kinds. Firſt, thoſe who are ſhort ſighted, and can diſtinctly diſcern Objects placed very near to the Eye. Secondly, thoſe who can diſtinctly diſcern Objects placed at a moderate Diſtance from the Eye. Thirdly, thoſe diſcerning Objects placed more Remote. And hence it is, that when Glaſſes are applied, ſome Perſons will be able to judge of an Object as appearing larger to them than to others; and the Spaces contained within their Bounds or Limits as apparently containing a greater or leſs Number of Parts.

28th. As the Eye one Perſon may have an Advantage

Of the Accuracy of Obſervations.

tage of that of another, ſo may the Faculties of the Mind, whereby Judgments is formed concerning the ſmalleſt Quantities of Duration or Time. Both of theſe Qualifications are applied in making the moſt perfect and accurate Aſtronomical Obſervations; and a Deficiency in either of them renders the beſt Obſervations imperfect. The following is one Inſtance of this kind.

29th. The Beginning of the Solar Eclipſe June 4th. 1769 was obſerved at Greenwich 15 Seconds of Time later by me than by another Perſon who uſed a Teleſcope magnifying about three fifths as much as that applied by me. But at the End of the ſame Eclipſe, my ſtricteſt Attention being kept up to the laſt Moment, the Obſervation differed but half a Second of Time from a Medium of the two other Obſervations, although they differed 5 Seconds of Time from each other. The Day before, in obſerving the Tranſit of Venus, I clearly ſaw the firſt external Contact of the Planet with the Sun's Limb ſooner, and the firſt internal Contact later than any of the other ſix Obſervers, thereby making the Planet's Paſſage over the Sun's Limb three Quarters of a Minute of Time longer than a Medium of the other Obſervations. Many other Inſtances of this kind might be added, ſhewing the uncertainty of the niceſt Obſervations.

30th. When two fine Threads are tightly ſtretched perpendicular to the Horizon, near the Plane of the Meridian, at the Diſtance of 8 Feet from each other, the Eye of an Obſerver ſteadily fixed, may perceive the Inſtant when the Sun toucheth the Threads without erring two Seconds of Time in a ſingle Obſervation, this is proved from original Obſervations in my *Practical Aſtronomy*. But, when a Teleſcope is uſed, of the ſame Length as the Diſtance of thoſe Threads and magnifying 80 times, it is hardly poſſible to pronounce the Inſtant when the Wire in the Teleſcope is touched by the Sun's Limb, without erring an 8th of a Second of Time in a ſingle Inſtance. As the apparent Semi-

nefs is 80 times increased by the Glass, the Contact might be observed to a 40th Part of a Second of time if there were no Imperfection in either the Sight, Hearing or Conclusion drawn from both; and therefore, in this Instance, a Defect of four fifths of the whole, ariseth through an Imperfection of the Senses to distinguish accurately concerning such minute Distances and swift Motions.

3dly. There is another Method of judging when either the Sun's Limb or a Star transits the Telescopic Wire, by observing it's Situation the Second of Time before it comes to and also the Second after it hath passed the Wire. Thus if the Beat of the Clock be when the Limb be short of the Wire by any apparent Space or Time and the next Beat when it is past the Wire by double that Quantity, the Transit is at a third part of a Second of Time after the first Beat of the Clock; but if it be doubtful whether the Limb be past half or a third of a Second at the last Beat, the Medium may be but a little more than a tenth of a Second of time different from the Extremes. Errors of this kind may be estimated thus: Suppose for Instance the Wire is touched by either a Star or the Sun's Limb, when it is more than a third, but less than half a Second of time, the Sum of one half and one third is 10 twelfths, the mean of which is 5 twelfths of a Second for the time of being on the Wire, which differs but a twelfth of a Second of time from either of the Extremes. But how difficult it must be to practise such a Method, may be judged of when it is but three fifths of a Minute of time the Star or Limb is passing from one Wire to another, from which the Observer hath but a little more than a quarter of a Minute of time, for making and writing down the Circumstances of one compleat Observation. The same Method is applicable in the Reading of the smaller subdivisions of Astronomical Instruments in Cases where the precise Degrees, Minutes, Seconds and Parts of a Second cannot be had; but this will avail little where other

Errors

Of the Accuracy of Observations.

Errors interfere either in the Construction or Use of Instruments.

3d. The small Space which is apparently described by the Sun's hundred Second of time, as that Space is viewed through the best Telescopes, and the quick Returns of the Beats of the Clock, cannot but make it extremely difficult to determine the Instant when the Limb is on the Telescopic Wire, without erring a fifth of a Second of time; and then, there will be three such Observations for assigning the Moon's Place in Right Ascension, namely, one for the Star's passing the Wire, one for the Sun, and another for the Moon; which three may produce an Error of three fifths of a Second of time, amounting to 9 Seconds of a Degree for the Motion of the Equator, and in assigning the Moon's Place thereby, an Error of 9 Geographical Miles, when they all fall on one Side of the Instant which is the Truth. There are commonly five Wires in the Telescope, at which, supposing them all placed right, an Error may arise after the same Manner, and of the same kind, and if they should happen to arise of a contrary kind, an original one may remain whose Quantity cannot be determined.

33d. If such an Error may arise from a Defect of the Senses and the Application of them in an Observation, how much greater may the Errors be arising from the Instruments used, and Inability to adjust them to the Truth? 1st, In counting the successive Order of small Portions of Time, the Mind with Difficulty determines to the fifth Part of a Second of Time. 2d, The Eye with great Difficulty discerns when the Telescopic Wire is within a Second of a Degree of the Sun's Limb. 3d, The Clock which is used may not have been exactly set to the Time intended to be kept in the Interval between the Observations. 4th, It may have Erred from it's Rate of going, and the Quantity of that Error may not be known. 5th, The Observer may err in judging of the Beats of the Clock as to the Instant they happen; or 6th, Of the Situation of

the Limb or Star on the Right or Left of the Wire at thoſe Times. 7th. The central Wire of the Teleſcope may not move in the true Plane of the Meridian. 8th. The other Wires may not be correctly Equidiſtant from the Central one. 9th. The Mark may not be correctly in the Plane of the Meridian, by which the Teleſcope is adjuſted. 10th. By Error in the Adjuſtment, the Horizontal Axis of the Inſtrument may not be ſet right by either the Spirit Level or Plumb Line. 11th. The Line of Collimation or Axis of Viſion may not be correctly in the Plane of the Meridian. 12th. The Teleſcope may become a little out of the Plane of the Meridian by its own Weight and Motion after it hath been adjuſted, this Error only may amount to ſeveral Seconds of a Degree. 13th. There may be a ſmall Refraction horizontally by Reaſon of Inequality in the Denſity of the Air, Eaſt and Weſt of the Meridian. 14th. There may be a ſmall Dilation or Contraction of the Sun or Moon's Limb according to the ſtate of the Atmoſphere. 15th. There may be a ſmall Error ariſing for want of knowing correctly the Moon's Diameter, or even that of the Sun, and another ſmall Error may ariſe concerning the Star's being exactly on the Middle of the Wire; and other Errors may ariſe, but where there no more;

34th. Of theſe fifteen Errors, ſome are found in Practice to amount to many Seconds of a Degree, ſome to a few, and others none that are viſible; whilſt in all of them, an Error of one Second of a Degree, would hardly be diſcernible by the niceſt Judgment and the ſharpeſt Eye, and this Error is nearly in the Direction of the Moon's apparent Path amongſt the Fixed Stars. Therefore, it may be uncertain (when the Moon's Place hath been obſerved by theſe Methods) how correctly it may therefrom be determined, to the amount of 15 Seconds of a Degree in Longitude; and this, in applying the Receſſion to the Longitude at Sea, would, at the Equinoctial, produce an Error of 7½ Miles.

35th. For the Reaſons before aſſigned, Tables of the Sun, Moon and Fixed Stars may be formed and tried,

Of the Accuracy of Obſervations.

tried, and found either to agree or diſagree within the Limits abovementioned, whilſt in Reality they differ conſiderably from Truth. If 15 Seconds of a Degree Error may be hid and undiſcovered, a Diſcovery of another 15 Seconds of a Degree Error on the ſame ſide of the Inſtant or Place of Truth, will make an Error of 30 Seconds of a Degree, and this produceth an Error of 15 Miles in Longitude. All this may happen without exaggerating the Errors, whilſt it is well known that greater ones may happen many times, to the Amount of double and even treble the forementioned Quantities.

X. *Of Deceptions in the Theory of Gravity.*

36th. Were theſe the only Difficulties to be overcome, the Limits of Errors ariſing through Imperfection in the Obſervations might be aſſigned; and the Obſervations being compared with genuine Predictions from a Theory, would ſhew to what Exactneſs it might be depended on; but it ſo happens, that there is Reaſon to ſuppoſe there may be other Circumſtances attending the Conſtitution of the Solar Syſtem beſide thoſe which Aſtronomers have hitherto introduced, that they are of an hidden and inſcrutable kind, and may extend their Influence to the Orbit of the Moon and produce a ſmall Effect in her ſeeming Irregularities. Theſe have been learnedly ſearched after by Mathematicians and Philoſophers, and Equations have been given for correcting them, but it happens, that ſometimes the Equations by different Perſons diſagree not only with one another, but with actual Obſervations; and it may be hard to decide whether the Error be greater in the Theory or the Obſervations themſelves, although made with the beſt Inſtruments and by the beſt Obſervers. Many of the ſeeming Inconſiſtences that appeared two Centuries ago againſt the true Solar Syſtem, have been ſince reconciled and that Syſtem proved to be under the general Laws of Matter and Motion; ſucceeding Times may bring to Light other hidden Truths, and prove that many things now taken for granted are

Of Deceptions in the Theory of Gravity.

erroneous by Quantities which are not too Minute to be diſcerned or perceived.

"The common Centre of Gravity of the Earth the "Sun and all the Planets is immoveable. For that "Centre either is at Reſt, or moves uniformly for- "ward in a Right Line. But if that Centre moved, "the Centre of the World would move alſo.

"The Sun is agitated by a Perpetual Motion, but "never recedes far from the common Centre of Gra- "vity of all the Planets. For Since the Quantity of "Matter in the Sun, is to the Quantity of Matter in "Jupiter, as 1067 to 1, and the Diſtance of Jupiter "from the Sun, is to the Semidiameter of the Sun, in "a Proportion but a ſmall Matter greater, the com- "mon Centre of Gravity of Jupiter and the Sun, will "fall upon a point a little without the Surface of the "Sun. By the ſame Argument, ſince the Quantity "of Matter in the Sun, is to the Quantity of Matter "in Saturn as 3021 to 1, and the Diſtance of Saturn "from the Sun in a Proportion but a ſmall Matter "leſs; the common Centre of Gravity of Saturn and "the Sun will fall upon a Point a little within the "Surface of the Sun. And purſuing the Principles "of this Computation, we ſhould find that though "the Earth and all the Planets were placed on one "Side of the Sun, the Diſtance of the common Centre "of Gravity of all from the Centre of the Sun, would "ſcarcely amount to one Diameter of the Sun. In "other Caſes, the Diſtance of thoſe Centres is always "leſs. And therefore ſince the Centre of Gravity is "in perpetual Reſt, the Sun, according to the various "Poſitions of the Planets, muſt perpetually be moved "every way, but will never recede far from that Centre. "Hence the common Centre of Gravity of the "Earth, the Sun and all the Planets is to be eſteemed "the Centre of the World. For ſince the Earth, the "Sun and all the Planets, mutually gravitate one "towards another, and are therefore, according to their "Powers of Gravity in perpetual Agitation, as the

"Laws

Of Deceptions in the Theory of Gravity.

" Laws of Motion requires, it is Plain that their move-
" able Centres cannot be taken for the immoveable
" Centre of the World. If that Body were to be pla-
" ced in the Centre towards which other Bodies gra-
" vitate most, that Priviledge ought to be allowed to
" the Sun. But since the Sun itself is moved, a fixed
" Point is to be chosen, from which the Centre of the Sun
" recedes least, and from which it would recede yet less if
" the Body of the Sun were denser and greater, and
" therefore less apt to be moved.

" The Planets move in Ellipses, which have their
" common Focus in the Centre of the Sun. Newton's
" Principia, Book 3. prop. 11. 12. 13.

37th. This is the Doctrine of the Solar System according to the Theory of Gravity. The Centre of that System is at Rest, the Centre of the Sun is sometimes but little less than a Semidiameter of the Sun or 16 Minutes from that Centre, at other times but little less than a Diameter or 32 Minutes from that Centre; the Difference of them is 16 Minutes, and this latter happens when the Planets in their Orbits are on the same Side of the Sun. If the Sun were of a much greater Density and Magnitude, he would be less liable to be moved, his Centre would remove at a less Distance from that Centre of Gravity; but now, the Centre of the Sun describes an intricate Curve arising from the Motions and Situations of the Planets in their Orbits. Lastly, the Planets move in Ellipses, whose common Focus is the Sun's Centre. If *a* be the immoveable Centre of Gravity, *b* the Place of the Earth, and *c* a Fixed Star at such a Distance as produceth no Parallax, and the Centre of the Sun be carried to *d*, the Earth by respecting the Sun's Centre will be carried to *e*; draw *edf* Parallel to *bac*, so will *f* and *c* appear to coincide. But if the Earth be at *g*, whether the Sun's Centre be at *a* or *d*, it is the same; therefore, the Sun and Fixed Stars will have the same angular Distances whether the Sun's Centre be in or out of the common Centre of gravity, and the Sun accompanied with all

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the other Planets, may not be improperly called a Planet moving round the common Centre of Gravity of all the Planets, in ſuch a Path and Time as is unknown.

38th. As the Sun hath a Motion round the common Centre of Gravity, it is probable that he moves likewiſe round his own Axis, although that Motion cannot be diſcovered. Sir Iſaac Newton ſuppoſed the Sun's Horizontal Parallax 10 Seconds and half, ſome Obſervations on the late Tranſits of Venus make that Parallax little leſs than 10 Seconds, others a little more than 8 Seconds, ſome 8 Seconds and half, others 9 Seconds. The leaſt of theſe gives the Sun's Diameter nearly 730000 Miles, but probable that Diameter may exceed 860000 Miles. Suppoſe it 800000 Miles and the Circumference is 2513280 Miles, round which, the Solar Spots, by actual Obſervations, have been ſuppoſed to move in 25d. 5h.; and therefore the Sun hath been ſuppoſed to revolve round his Axis in that time. At this Rate the Suns Equator would revolve 4154 Miles per Hour, which is between 4 and 5 times as ſwift as the Rotation of the Earth. But no Inference can be drawn concerning the Sun's Rotation by the Solar Spots. More than 14 Years ſince I diſcovered by Obſervations that thoſe Spots are not near the Sun's Surface, and Publiſhed this Account of them in the Public Papers before any other Perſon. The mean Diameter of the Sun is nearly 32 Minutes or 1920 Seconds of a Degree, and ſuppoſing his Diameter 800000 Miles, a Second of a Degree on that Diameter would be 416 Miles and 2 Seconds 832 Miles. One Second of this Error may ariſe through Imperfection of the Senſes and Judgment, another through a Defect in the Inſtrument uſed, or there may be a ſmall Refraction horizontally contrary to the received Opinion; ſo that the Sun may be an oblate Spheroid flat towards the Poles, with its two Diameters more than 800 Miles different in Length, with its Axis differently ſituated from the preſent received Opinion, and this producing

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Of Deceptions in the Theory of Gravity.

an Error in ſeveral of the Planets Places, may be the Cauſe of ſuch other ſmall Inequalities as cannot eaſily be allowed for. In meaſuring the horizontal Diameter, the Refraction is ſuppoſed Nothing; in the vertical Diameter, allowance is made according to the ſuppoſed Refraction at the upper and lower Limbs, poſſibly there may be Error in both, and two or three Seconds more Error, would make the Sun's two Diameters which terminate at the Equator and the Poles, have a much greater Difference, and it is not eaſily to be determined what ſmall Effect this might have on the central Parts of the Solar Syſtem, or whether it might not affect the Orbit of the Moon and Earth. But whether there be any ſuch or not, the ſmall Errors before mentioned ariſing in the Obſervations, and from them entering into the Predictions of the Places of the Sun, Moon and Fixed Stars, are real and without a Fallacy; to which if there be added the other Errors, though ſmall, that may be in the uſual Equations and Allowances deduced from the beſt Theories, it will be hard to conceive how the Moon's Place can be predicted truly to a Second of Time, although deduced from the beſt Obſervations and Calculations. To ſuch a Degree of Exactneſs, Obſervations can be made on Land only, where the largeſt Inſtruments can be uſed in the ſteadieſt Manner. At Sea Inſtruments that are portable only can be applied, and theſe are often rendered leſs ſerviceable by the Motion of the Ship. But of all the Angular Inſtruments that have been uſed at Sea, none hath anſwered ſo well for general Uſe as Hadley's Octant and Sextant, and therefore it hath been of great Uſe in finding the Longitude at Sea, by the Moon's Diſtance from the Sun and Zodiacal Stars.

XI. *Of Deceptions in Hadley's Sextant.*

39th. This Inſtrument, as it is uſually made for Aſtronomical Obſervations at Sea, is not leſs than one Foot nor more than two Feet Radius, and becauſe each nomical Degree on the Arch or Limb is but half a Degree of the Circumference to the Radius of the Inſtrument

Of Deceptions in Hadley's Sextant.

ſtrument, it therefore follows, that a Minute of a Degree on the Arch of the Sextant is nearly a thirty third Part of the tenth of an Inch when the Radius is a Foot; and nearly a twentieth Part of the Tenth of an Inch when the Radius is 18 Inches; the Smallneſs of which Parts, render them ſomewhat difficult to be diſtinguiſhed, although the Vernier or Nonius Diviſion be adapted for making them legible. But notwithſtanding this, when the Diviſions are cut fine and in the moſt correct Manner, and the ſtricted attention is employed in reading them, they may be judged of to near a quarter of a Minute of a Degree; but ſuch Diviſions are rarely to be met with; and to be able to judge to half a Minute in a Foot Sextant and to a third Part of a Minute in an 18 Inch Sextant is the beſt that can be expected in ordinary Practice by the beſt Inſtruments and Obſervers.

40th. On account of theſe Limits, 10 Seconds of a Degree are not to be diſcerned on the Arch of the Inſtrument however accurately it is divided, and this alone will produce an Error of 5 Miles in Longitude at the Equinoctial; but if 20 Seconds be admitted for ſuch an Error in reading, the Error in Longitude may be 10 Miles at the Equinoctial; and this only through an Overſight or Error in reading the Diſtance of the Sun and Moon, when it hath been obſerved. A ſecond Error to the ſame Amount, may ariſe in ſetting the Beginning of the Index to the Beginning of the Sextant; and a third Error of the ſame Quantity, by an imperfect Junction of the Object as formed on the quickſilvered and unquickſilvered Glaſſes. If theſe three Errors only amount to but 10 Seconds each, the whole is half a Minute of a Degree; this may be the Caſe and each of the three Errors, with the greateſt Difficulty (if at all) perceived by the Senſes. This Error, in taking the Moon's Diſtance from the Sun or a Zodiacal Star, will be 15 Miles at the Equinoctial. If each of theſe Errors is 20 Seconds, the Error of Longitude will be 30 Miles: Theſe are but a few amongſt the many ſmall Errors that may ariſe in the Conſtruction, Adjuſtment and

Application

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Application of this valuable Instrument, the greater Part of which may be scarce discerned by the Senses, and therefore, they may frequently come together, and make an Error which is considerable. For, if the Instrument be so framed as to yield in an extremely hot or cold Climate; if there be any Error in the Divisions; any shake at the Centre; if the Glasses are not Parallel Planes; and if they are bent through Accident; if they are not exactly adjusted; if the Eye is not at the proper Place; if the Objects are not united at the proper Place on the small Mirror; and the Limbs of the Sun or Moon be imperfectly defined; twelve Errors may arise in a single Observation, each of which may amount to 5 or even to 8 Seconds of a Degree; one half of which Error will amount to 30 Nautical Miles, and it is extremely hard to determine how many of these may be in every Observation.

XII. *Of Time kept by Clocks and Watches.*

421st. Of all the Instruments that have been invented for keeping Equal Time, none have been found to answer so well as the Pendulum Clock for Land and the small Watch for Sea. The Perfection of the Clock depends almost wholly on the invariable State of it's Pendulum, the other Parts of the Machine being properly made according to Art; this Conclusion is drawn from Experiments, amongst which, I take the following to be a remarkable one. Toward the End of the Year 1775, I had set up two Regulators in my Apartment at the Distance of 12 Feet from each other, and adjusted them as accurately as possible to mean Solar Time, and so they went on, sometimes one varying at other times the other, 'till the Beginning of the Year 1776, when the great Frost set in accompanied with deep Snow, the Thermometer stood many Degrees below freezing a long time, and the Cold was said to be as intense as it had been in Memory of the oldest Man then living. I continued to readjust the Clocks after the Frost began, hoping to get a few Meridian Altitudes of the Sun and try the Effect of Refraction at that time,

Of Time kept by Clocks and Watches.

but could take none; and finding the Clocks were ſo regulated that they began to keep together, I obſerved how long that happened, and found it 5 Days that the Beats of the one agreed with the Beats of the other, without differing any Part of a Second of time diſtinguiſhable by the Ear. Before the Froſt ended, the Beats began to differ; and a Day after that, the Thermometer roſe a little, and then came on the Thaw. Whilſt the Clocks went on beating together, I conclude their Pendulums were frozen and thereby incapable of lengthening through Heat or ſhortning through Cold; and that the other Obſtructions amongſt the Wheels were but ſmall in compariſon with the Effects of Heat or Cold. Theſe were Pendulums fixed by Nature to the ſame time of Vibration. Gridiron Pendulums are artificial ones intended for the ſame Purpoſe, and are very efficacious for keeping the Centre of Oſcillation of the Pendulum at the ſame Diſtance from the Centre of Motion; but in other Pendulums, were this Proviſion is wanting, the Irregularities will be greater, but ſeldom ſo much as cannot be eaſily diſcovered by frequent Compariſions with obſerved Meridian Tranſits of the Sun and Stars.

42d. As the Pendulum Clock is the beſt Time Keeper on Land, ſo is the Pocket Watch of great Uſe at Sea, when made in the beſt manner, and it is carefully kept in the Pocket under nearly the ſame Degree of Heat. The better Sort of Watches beat near 18000 Beats in an Hour, or 300 Beats in a Minute, or 5 Beats nearly in a Second of Time; this is nearly that ſmall Portion of Time, to leſs than which (as hath been before concluded) it is uncertain to aſſign the Inſtant of an Aſtronomical Obſervation.

XIII. *Of Longitude by Help of Pocket Watches.*

43d. The Uſe of the Watch is to keep a certain Space of Equal Time truly, whether it be long or ſhort, and to ſhew the intermediate Subdiviſions by the Minute and Second Hands; this Time may be either Siderial or Mean Solar Time, but the latter is beſt, being moſt eaſily converted into Solar Time, by which the Obſervation

Of Longitude by Help of Pocket Watches.

vation is reckoned. Here, the apparent Revolution of the Heavens is 24 Hours of Time, therefore a Degree of Longitude is 4 Minutes of Time, a Minute of Longitude 4 Seconds of Time; alſo, a Minute of Time is 15 Minutes of Longitude, and a Second of Time is 15 Seconds of Longitude.

44th. Conſequently, Watches, when they keep Time truly, may be applied in finding the Longitude either by Land or Sea. For Inſtance, if ſuch a Watch be ſet to Mean Solar Time for any Meridian from which a Perſon departs, whilſt it keeps Time truly, it will ſhew the Time of Day or Night for that Meridian, although it be removed to any other Meridian, and the Reduction to Solar Time is eaſily made by the Equation Table; but the Solar Time at the Place removed to is eaſily found by Obſervation, and this compared with that inferred by the going of the Watch, turned into Degrees, gives the Longitude from the Place removed from; Eaſt when the Time by the Watch is leaſt, otherwiſe Weſt. Thus Longitude is reckoned to 180 Degrees both Eaſt and Weſt, and this counts quite round the Earth's Globe.

45th. The ſame Method is to be obſerved, if a Spring Clock or any other Time Keeper be uſed; but it is to be noted, that if there be any Error in the going of ſuch a Machine either quicker or ſlower than Truth, or the gaining or loſing be not known, the Concluſion will be erroneous; and therefore, in the Uſe of ſuch Machines, the greateſt and moſt conſtant Care ſhould be taken to diſcover their Imperfections, and the Effect of any Accidents they may be ſubject to; otherwiſe, the Longitude found by their Uſe may be uncertain.

XIV. *Of the Moon's Place obſerved at Sea.*

46th. By the Moon's Place is here meant, her Longitude or Situation in the Ecliptic as viewed from the Earth, which Place (as hath been before noted) ſhe is continually receding from, nearly in the Direction of the Ecliptic from Weſt to Eaſt, amongſt the Fixed Stars. The Inſtrument by which the Moon's Place is

Of the Moon's Place obſerved at Sea.

now commonly taken, is the Sextant or Quadrant before deſcribed, and this is effected by bringing the Moon's enlightned Limb either to a Junction with the Sun, or with the Centre of a Zodiacal Star. In the Obſervation by Day, the Suns neareſt Limb is brought to the Moon's neareſt Limb; but by Night, the Moon's neareſt or remote Limb is brought to the Star, and Addition being made of the Sun's Semidiameter, and either Addition or Subtraction being made of the Moon's Semidiameter, gives nearly what would be the obſerved Diſtances of the Centres, could that be taken. In the making of this Obſervation either by Sea or Land, the greateſt Care and Diligence ſhould be employed; the Inſtrument adjuſted as correctly as poſſible, the Limbs made to touch each other as exactly as poſſible, and apparently on the right Part of the Little Mirror of the Inſtrument; becauſe an Error of 2 Minutes of a Degree in this Diſtance, will (in Application to the Longitude by the Moon's Place) produce an Error of 60 Miles at the Equinoctial, and in Proportion for leſſer Errors. The Error by uncertainty in reading the Inſtrument, may not be leſs than 15 Seconds of a Degree, and nearly as much Error may ariſe concerning the Junction of the Limbs of the Sun and Moon or Moon and Star, when the Obſervation is made under the moſt favourable Circumſtances, with the beſt Inſtruments and by the moſt acute Obſervers. Theſe together would make an Error of 15 Miles in Longitude at the Equinoctial. I have found, by many careful Obſervations, that this Accuracy can be attained to, by meaſuring the Sun's Diameter with the Inſtrument, but not without expoſing my Eye and keeping up ſuch an Attention for Judgment, as is by no Means to be recommended to or expect from Obſervers in general; who may find it ſomewhat difficult to obſerve and read to a Minute of a Degree, which will produce an Error of 30 Miles in Longitude at the Equinoctial. But frequently the Junction of theſe Errors will be leſs, when they happen to counter-act each other.

47th.

Of the Moon's Place observed at Sea.

47th. The Moon's Diſtance being thus taken, either from the Sun or a Zodiacal Star, her Longitude near the Ecliptic is determined ſomewhat near the Truth, becauſe the Longitude of the Sun or of the Star is known at the ſame time. This Place is not correctly determined becauſe the Sun may be affected by Refraction, and a little by Parallax in Altitude; but the Moon may not only be affected by Refraction, but very much by Parallax in Altitude; and both of theſe muſt be allowed for, before the true Diſtance of Centres can be known.

XV. *Of Refraction in Altitude.*

48th. When a Ray of Light comes from a luminous Body and paſſeth through a Medium that is of equal Denſity every where, the Ray proceeds forward in a ſtraight Line throughout ſuch a Medium, from the Beginning to the End of it, or 'till it is abſorbed; this is certain from the Principles of Optics. But if the Medium be of unequal Denſity, the Ray will ever incline toward that Part of the Medium which is moſt denſe. The Atmoſphere ſurrounding the Earth and Sea (the principal Part of which reaches to the Height of 50 Miles) being a Fluid, muſt neceſſarily be more denſe at Bottom than at Top, on two Accounts; firſt, becauſe the upper Parts preſs on the lower by their Weight and thereby condenſe them; ſecondly, becauſe the groſſer and more weighty Particles of the Atmoſphere compoſed of Air and groſs Vapours, do of Courſe deſcend lowermoſt. Conſequently, a Ray of Light coming from the Sun, and paſſing through the Atmoſphere, in any other than a Perpendicular Direction, muſt paſs through a Curve which is concave toward the Earth; this Effect is greater when it comes horizontally from either the Sun, Moon or Stars, and at the Zenith it diminiſhes to Nothing. From this Cauſe, Refraction in Altitude ariſeth, its Quantity near the Horizon is 31 Minutes in Summer and 35 Minutes in Winter for London, but different at other Places. Near the Equinoctial

Equinoctial it has been found 27 Minutes but much greater in high Latitudes toward the Pole.

49th. Let *a* be the Place of a Spectator on the Earth, *bc* an Arch of the Top of the Atmoſphere, *d* the higher and *e* the lower Point of the Sun's Diameter, *f* the Place where a Ray of Light coming from *d* enters the Atmoſphere, and *g* were another enters it coming from *e*; then will *d* appear at *b* and *e* at *i*; and the Sun's real Diameter *de* will appear to be *bi*, in which Caſe, *i* is refracted more than *b*; and therefore, near the Horizon, the Sun or Moon's Centre will be refracted near 4 Minutes of a Degree, more than the upper Limb; which in taking the Diſtance of the Sun and Moon or of Moon and Star, when they are in or near the ſame vertical Circle, may produce an Error in Longitude of near 120 Miles. But this Error will be diminiſhed to 4 Miles in Altitudes not exceeding 10 Degrees, and in greater Altitudes it will become leſs, and in the Zenith nothing.

50th. If there be no other but this Vertical kind of Refraction (as is the general Opinion) and the Refraction for the upper Limb be taken for that of the Centre, the Diſtance of Centres will thereby be erroneous: 4 Seconds of a Degree as high as 15 Degrees of Altitude, and under it more, but above leſs, and three times that Quantity of Error is hardly to be preceived in Practice either on the Inſtrument or in the Heavens. Therefore Corrections to a ſmall Part of a Second for Refraction, or even to a few whole Seconds themſelves, when they embarras the Calculation with Difficulties, may be neglected. When the Sun and Moon have equal Altitudes, and are both near to each other and to the Horizon, the Refraction of their Centres will be equal to that of their Limbs; becauſe, in that Situation, their Centres and the Points in the Limbs to be joined by the Sextant, have the ſame Altitude; conſequently, when the Luminaries have ſuch Equal Altitudes, their Diſtances are leaſt affected by Refraction. But when they are in the ſame vertical Circle that Effect is greateſt. Part of this Error

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Error in the Practice, ariſes from the Difference between the Refraction of the Centre and the upper Limb of the Luminary which is lowermoſt in that vertical Circle, which at the Altitude of 10 Degrees, for the Sun and Moon, is but 7 or 8 Seconds of a Degree, and that is imperceptible. When a Star is lowermoſt in that vertical Circle this Error wholly vaniſhes; and therefore, in greater Altitudes, this Effect will be leſs, and no conſiderable Error can ariſe from Refraction, by taking the Refraction of the Limbs for that of the Centres, under theſe Circumſtances.

XVI. *Of the Moon's Parallax in Altitude.*

51ſt. The Moon's Parallax in Altitude is the Angle by which ſhe is elevated above the true Horizon, when ſhe is in the apparent or viſible Horizon of the Place of Obſervation; and this Angle, generally, is more than 54 Minutes of a Degree, when the Moon is moſt remote from the Earth, and leſs than 62 Minutes, at her neareſt Approach to the Earth; and according to all the intermediate horizontal Parallaxes, ſo will her horizontal Diameter be propotioned, from the Apogee to the Perigee of her Orbit. For Inſtance, if by Obſervation and Computation, when the Moon's horizontal Parallax is 61m. 15s. her horizontal Diameter meaſures 33m. 23s. and again, when her horizontal Parallax is 54m. 13s. her horizontal Diameter meaſures 29m. 35s., then, when her horizontal Parallax is 57m. her horizontal Diameter will meaſure 31m. 16s. For as the Moon's horizontal Parallax is enlarged by her neareſt Approach to the Earth, ſo is her Diameter enlarged as viewed from the Earth, by the ſame Cauſe. Hence the Moon's horizontal Diameter and her horizontal Parallax become a little different from each other, at the Noon of every Day; and becauſe the Moon is nearer to an Obſerver on the Earth's Surface, when ſhe is elevated than when ſhe is in the Horizon; at any Altitude, her Diameter meaſures a little more than her horizontal Diameter at the ſame Diſtance from the Earth.

52d.

Of the Moon's Parallax in Altitude.

52d. If *a* be the Earth's Centre, *b* the Zenith, *c* a Place on the Earth's Surface in the Line *ab*, *ad* the true Horizon paſſing through the Earth's Centre, *e* the Moon, the Angle *bae* her true Coaltitude as ſeen from the Earth's Centre, and the Angle *bce* her obſerved Coaltitude as ſeen from the Earth's Surface, then is the Angle at *e* the Difference of thoſe Angles, and as *ae* to *ac*, ſo is Radius to Sine of the Angle *e*. Therefore, as Radius to Coſine of the Moon's obſerved Altitude, ſo is her horizontal Parallax, to her Parallax in Altitude. And as Coſine of her Altitude to Radius; ſo is the Difference between her Diameters as they meaſure reſpectively in the Zenith and Horizon, to the Quantity whereby the horizontal Diameter is augmented at that Altitude. By theſe Proportions, a Table may be formed not only of the Moon's Parallax in Altitude, but alſo of the Number of Seconds of a Degree, whereby the horizontal Diameter is to be augmented, ſo that her Diameter may agree with Obſervations; theſe are called ſmall Parts for the Moon's Diameter, and their halfs for her Semidiameter in Altitude.

XVII. *Of the true Diſtance of Sun and Moon's Centres.*

53d. By the true Diſtance of the Sun and Moon's Centres, is here meant, the Degrees, Minutes and Seconds of a great Circle in the Heavens, which would appear to join their Centres, could an Obſervation be made by a Perſon at the Centre of the Earth; this will be a little different from that Arch of a great Circle, which appears to join thoſe Centres, to a Perſon on the Earth's Surface; and their Difference is called the Effect of Parallax in Diſtance. In Order to the better underſtanding of this moſt neceſſary and uſeful Correction, and get the true Diſtance of the Centres, it is proper to conſider the Effects of Refraction and Parallax as they together produce an Error in the Diſtance of the Centres. But firſt it is proper to conſider the Effect of the ſpheroidal Figure of the Earth.

54th. When an Obſerver hath taken the Altitudes of the Suns lower Limb, and added thereto the Semi-

diameter

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diameter, but ſubtracted the Dip for the Elevation of his Eye above the Horizon of the Sea, he commonly ſubtracts alſo the Refraction in Altitude and then concludes the Remainder is the true Altitude; and the like for the Moon or a Star; but that this is not correctly the Caſe, may be ſeen in either my Treatiſe of *Aſtronomy* or *Navigation*. The Error hereby produceth but little Effect in the Quantity of either Refraction or Parallax when they are the only Things wanted, and therefore, in uſual Caſes, this Correction may be neglected as inconſiderable. And ſeeing that the horizontal Refraction is but a little more than half the horizontal Parallax, and upward the Refraction in Altitude decreaſeth in its Quantity much faſter than the Parallax in Altitude, their Difference will be the Moon's Elevation in a vertical Circle, to bring her to the Altitude where ſhe would be viewed from the Centre of the Earth. But for the Sun's Parallax in Altitude (if his horizontal Parallax be 9 Seconds) it will be but 8 Seconds at 20 Degrees Altitude, 7 Seconds at 35 Degrees Altitude; 6 Seconds at 50; 5 at 55; 4 at 60; 0 at 90; ſo that the Suns Parallax in Altitude will not only be very ſmall and inconſiderable, but always leſs than the Parallax in Altitude, and vaniſhing with it at the Zenith.

55th. From theſe Conſiderations, an eaſy Method ariſeth of ſolving this Problem, thus. Let *ab* be an Arch of the Horizon, *c* the Zenith, *ad* the Altitude of the Sun's Centre cleared from Semidiameter and Dip of Horizon, *be* the Altitude of the Moon's Centre cleared from Semidiameter and Dip of Horizon; and becauſe Refraction apparently elevates, but Parallax in Altitude apparently depreſſeth the Sun and Moon, let *df* be the Difference between the Refraction and Parallax of the Sun, and *eg* that of the Moon; ſo will *de* be the obſerved Diſtance of the Centres, and *fg* their true Diſtance cleared from Refraction and Parallax.

56th. In the oblique angled ſpherical Triangle *cde*, *cd* being the Sun's Co-altitude, *ce* the Moon's Co-altitude, and *de* the obſerved Diſtance of the Centres, find the Angles *cde* and *ced* by any of the uſual Spherical

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Theorems; make *dh* Perpendicular to *fg* and *gi* Perpendicular to *de*. The Circumſtances attending this Problem are theſe; the two Co-altitudes are each of them always leſs than 90 Degrees, and the Diſtance ſeldom leſs than 20 Degrees, but never exceeding 120 Degrees. At 8 Degrees Altitude there may be a Change of the Quantity of Refraction from Summer to Winter of half a Minute of a Degree, which may be preceptible to the Eye of the Obſerver and legible on the Sextant; at 20 Degrees Altitude, this Effect becomes doubtful, being but half that Quantity; *f* will always be lower than *d* becauſe Refraction elevates but Parallax depreſſeth, and the Refraction is always greater than the Sun's Parallax in Altitude; but *g* will be higher than *e*, becauſe the Refraction will always be leſs than her Parallax in Altitude; when *d* is not leſs than 8 or 10 Degrees Altitude, *df* will but little exceed 5 or 6 Minutes of a Degree, therefore the Angle *gfe* will be very nearly equal to the Angle *edc* in greater Altitudes, and one may be taken for the other without perceptible Error in many practical Caſes; the Angle *ced* is correctly found by the three Sides; *df* is correctly known according to any ſuppoſed Refraction and Parallax; the little Triangles *dfh* and *gei* are inſenſibly different from Right Angled Plane Triangles, whoſe Sides are Right Lines and Angles Plane Angles; the Spherial Angle *gfc* is inſenſibly different from the Plane Angle *dfh*, and the Spherial Angle *dec* inſenſibly different from the Plane Angle *gei*; *ge* becomes a Maximum or the longeſt that it can be when *e* is between the Limits of 10 and 20 Degrees Altitude, and even then amounts to but 52 Minutes of a Degree; at the Horizon it amounts to but half that Quantity near the Equinoctial, and above 20 Degrees Altitude it diminiſhes to Nothing in the Zenith; *fh* is the Coſine of the Angle *f* to the Radius *df*; *ei* is the Coſine of the Angle *e* to the Radius; *ge*, *di* is very nearly equal to *gh*; for theſe are nearly equal to each other, namely *dg* and *di*, becauſe *gi* is always ſmall being compared with *di*; *dg* and *gh*, becauſe *gh* is ſo very

ſmall

Of the true Diſtance of Sun and Moon's Centres.

ſmall being compared with *gb*, thefore *di* and *gb* become very nearly equal. Hence, when the Angles *e* and *f* are acute, that is leſs than 90d. *ei* is ſubductive and *fh* additive, to get the true Diſtance of the Centres; but if the Angles are otherwiſe, a contrary Effect takes Place, that is Addition is uſed inſtead of Subtraction.

57th. If a farther Correction be required, by ſo much as *dg* exceeds *di*, will *ei* be too much; and by ſo much as *dg* exceeds *gb*, will *fh* be too much, but ſuch a final Correction will ſeldom be wanted, except the Diſtance be ſmall and the Moon's Altitude between 10 and 20 Degrees, whereby *ge* becomes a Maximum, in many other Caſes, the true Diſtance of Centres very nearly comes out without farther Correction.

58th. But this Method extends farther; in joining the Sun and Moon's Limbs by the Sextant, the true Diſtance of Centres is not ſtrictly to be inferred from this Correction and the Addition of their correct Semidiameters, except the Altitudes are equal. Let *a* be the Zenith, *b* the Sun's Centre, *c* the Moon's, when their Altitudes are equal; join their Limbs by the Sextant in the Points *d* and *e*, and the Effect of Refraction and Parallax in Altitude will be the ſame as at the Centres. But ſuppoſing *f* and *g* the Points in the Limb which are ſo joined and *h* the Moon's Centre, the Refraction and Parallax at *f* and *g* (which are the Points obſerved) will differ from that at *b* and *h*. Make *hl* Perpendicular to *hk*, then is *af* the true Co-altitude to the Point *f*, and *ag* to that of *f*. And becauſe the Angles *abh* and *abb* are Spherical Ones and known, and the Angles *fbd* and *ghl* are Plane Angles of the ſame Quantities to the Radii of the Sun and Moon; make *mf* and *gn* each Parallel to the Horizon, ſo will *bm* be the Coſine of the Angle *fbm* to the Radius *bf*, and *bn* the Coſine of the Angle *kbg* (or rather its Supplement) to the Radius *gb*; and this will give the Points *m* and *n* in the vertical Circles, from which the corrected Refraction and Parallax in Altitude is to be reckoned. In this Caſe, the two Co-altitudes *am* and *an* or rather their equals *af* and *ag*, with *fg* the obſerved Diſ-

tance of the Limbs, are to be worked for, to find the Effect of Parallax in Diſtance; and then, the Sun and Moon's corrected Semidiameters are to be reſtored, to get the true Diſtance of Centres.

59th. From the ſame Conſtruction ariſeth an eaſy Method of making the Obſervation, ſo that the whole Calculation may be made for the true Diſtance of the Centres at once. For ſince the Diſtance of the Limbs is had as accurately as it can be taken with the Sextant, that Side of the Triangle is always certain and given. But for the other Sides, if the Refractions for the Points of Contact were taken for thoſe of the Centres of the Sun and Moon, at the Altitude of 10 Degrees, the Difference would be but 3 Seconds of a Degree; and the Parallax in Altitude for the Centre, would exceed that for the Moon's upper Limb (at that ſame Altitude) but 3 Seconds of a Degree; at 30 Degrees Altitude, this Difference becomes 8 Seconds; at 50 Degrees it is 12 Seconds; at 70 Degrees it is 15 Seconds and near the Zenith it is 16 Seconds. The Difference ariſing from Refraction of the Centre and the upper Limb, in approaching toward the Zenith will be leſs. But to expunge both of theſe Errors, obſerve the following Rules. 1ſt. If the Luminaries have nearly equal Altitudes, the Refraction and Parallax in Altitude for their Centres, will often be that for their Points of Contact. 2d. If they are on the ſame Side of the Zenith, that for the Contacts muſt be, for the loweſt of them 16 Minutes of a Degree higher than the Centre; but for the higheſt, 16 Minutes lower than it's Centre. 3d. If they are nearly in the ſame Meridian, but on different Sides of the Zenith, that for the Contacts muſt be for 16 Minutes higher than the Centre of each of them. 4th. If they are in any other oblique Poſition; firſt take a View and form a Judgment from that Poſition, how many Minutes of a Degree the Point of Contact at the Limb of the one may be higher than its Centre, or the Point of Contact at the Limb of the other below its Centre; this, in uſual Caſes, may be done to

a

Of the true Diſtance of Sun and Moon's Centres.

a Quarter of the Semidiameter; and having taken the two correſpondent Co-altitudes, by them and the obſerved Diſtance of the Limbs, find the true Diſtance of the Limbs cleared from Refraction and Parallax, to which add the two true Semidiameters of Sun and Moon for that Altitude, and it will be the Diſtance of Centres required.

60th. From theſe Principles it follows, that in many Obſervations the Effect of Refraction and Parallax may be computed from the obſerved Diſtance of the Limbs and the Co-altitudes of the Centres, without ſenſible Error; and this Method is ſtill leſs imperfect when a Star is uſed inſtead of the Sun, becauſe it hath no apparent Semidiameter. And if the whole of this Correction be made, there is another ariſing from the Earth's Figure, a fallacious Horizon unequally affected by Refraction and Circumſtances not eaſily to be explained in a few Words, but what may be ſeen at large both in my Treatiſe of *Practical Navigation* and *Practical Aſtronomy*.

XVIII. *Of a Combination of Errors.*

61ſt. When an Error happens to be in either an Aſtronomical Obſervation or Computation; if it be more than the Truth, it is uſual called *plus*, but if leſs than the Truth *minus*; and all Errors will belong to one or the other of theſe two kinds, whether they are diſcernible or not. Hence it follows, that if there be but one Error in either an Obſervation or Computation, ariſing imperceptibly, whilſt it is ſuppoſed there is none, the Error cannot be known. This will be the Caſe with Reſpect to one Error only, to which if there be added a viſible Error of the ſame kind, it may be doubled or perhaps trebled; but if of another kind, it may be deſtroyed, and the Truth remain undiſcovered. If ſuch an Error ariſe once whilſt the Mind is ſtrictly ſearching after Truth, it is to be expected a ſecond and a third time, whilſt the ſame Attention is employed, and therefore there is no Probability that a Medium of ſeveral ſuch Obſervations will determine the Truth itſelf.

62d.

Of a Combination of Errors.

62d. When two or more Errors happen, each of them may be ſuppoſed to ariſe either through Imperfection of the Senſes, the Mind or Judgment; and therefore, when two or more Obſervations or Computations are made, and the Medium of them taken; the Truth is ſtill concealed, and an approach is made rather to what is taken for Truth, by imperfect Senſation or Judgment, and the Error is ſtill hid and undiſcovered. For Inſtance, if I meaſure the Sun's Semidiameter three times, and find it 15, 16 and 17 Minutes, whilſt through Imperfection either in Senſation, Judgment or the Inſtrument, each of the three meaſures is 15 Seconds more which cannot be diſcerned; the Medium will be 15 Minutes, but the Truth 15 Seconds more, here will be an Error undiſcovered.

63d. Conſequently, in a Combination of Errors, the Truth is ſo concealed that it never can be brought forth with Certainty by the uſual Method of taking a Medium; not the Truth itſelf, but a Quantity which according to Probability, differs from the Truth by ſuch a ſmall Difference as may be aſſigned; and when that Probability hath been admitted, there are other Probabilities reſpecting the ſame Combination that may take Effect, although leſs favourable, but it is quite uncertain when one or the other of them will happen. In taking their Medium, it is always ſuppoſed that the plus and minus Quantities do nearly counteract each other, which is contrary to this Doctrine. For, if by a Medium of one Sett of Obſervations (when the Error by plus and minus is nearly equal) the Truth is found within 5 Seconds of a Degree, in a Courſe of Events it may be expected, that by another Sett of Obſervations, the Truth may not be approached by a greater Quantity; and ſometimes a conſiderable Part of the Errors of one kind only may ſo fall on one Side of the Truth together, that the Medium may be very erroneous although it be taken for near the Truth itſelf.

XIX. *Of finding the Longitude at Sea.*

64th. There are four different Methods whereby the

Of finding the Longitude at Sea.

the Longitude at Sea may be diſcovered; and although they are not all attended either with the ſame Exactneſs or Convenience, they have all been found practicable and uſeful in Navigation.

Firſt Method of finding Longitude at Sea.

The firſt Method is by keeping as accurate an Account as poſſible of the Courſe and Diſtance ſailed, taking the Latitude as correctly and often as Opportunity permits, making due Allowance for Variation, Leeway, and Currents, and then determining the Difference of Longitude from one Day to another; all which is the Subject of *Practical Navigation*, and is fully treated on in a Treatiſe on that Subject lately Publiſhed, as an Appendage to this Work.

Second Method of finding Longitude at Sea.

The ſecond Method is by Help of a Time Keeper, which, it is ſuppoſed, will keep Equal Time during a Voyage, without erring 4 Minutes of Time if the Longitude is to be found thereby, or without an Error of 60 Nautical Miles at the Equinoctial, and in Proportion for a greater or leſſer Error in Longitude. In this Method, it is beſt to ſet the Time Keeper to Mean Solar Time, for the Meridian of ſome Place from which the Ship ſails, and having found the Solar Time at the Place where the Ship is come to, compare it with the Solar Time for the ſame Inſtant as inferred from the Watch, and the Difference is the Longitude of the Ship in Time from the Place where the Watch was ſet, if it hath kept mean Solar Time during the Voyage. But if it has gained or loſt no more than a Second of Time per Day for a Month, the Error thereby will amount to 15 Miles in Longitude at the Equinoctial, and in Proportion for any greater or leſſer Error that may ariſe, either in the Time Keeper or judging what it may have gained or loſt in that Interval.

Third Method of finding Longitude at Sea.

The third Method is by Help of an Ephemeris in which the Moon's Place (or what amounts to the ſame Meaning, her true Diſtance either from the Sun or Zodiacal

diacal Stars, as ſuppoſedly viewed from the Earth's Centre) is ſet down for the Beginning of every Day, alſo for ſome equal Part of a Day, throughout the Year. To which muſt be added, firſt a correct Method of obſerving the Place of the Moon in the Heavens, to determine the exact Solar Time at the Place for which the Prediction is made; ſecondly, a correct Method of determining the Solar Time at the Ship for the ſame Inſtant; and the Difference of theſe two Times turned into Degrees, gives the Longitude of the Ship from the Place for which the Place of the Moon was predicted. In this Method, the Motions of the Celeſtial Bodies are ſuppoſedly regulated by that immutable Law of Nature which peculiarly belongs to them, and which (if Aſtronomical and Phyſical Enquiries be ſufficient for enabling Men to compute its Effect) being known, the predicted Places of the Sun, Moon and Fixed Stars, for Time to come, are affected with but very ſmall Errors, and therefore render them of Uſe in their Application to the Longitude at Sea. But after all that can be done by this Method, ſeveral ſmall Errors will unavoidably introduce themſelves into the Practice, which can never be wholly exterminated, ſome of which have been particularly conſidered in the foregoing Pages, and when theſe or other ſmall Errors affect the Practice of this Method, the Concluſion will be accordingly; and to what ſuch Errors may amount in uſual Caſes, is left to the judgment of every judicious Perſon to determine, that no Attention, Care or Ingenuity may be wanting, when the moſt perfect Inſtruments that can be had are applied, to make this Method as ſucceſsful as poſſible.

Fourth Method of finding Longitude at Sea.

The fourth Method is by Variation Charts of the Magnetic Needle.

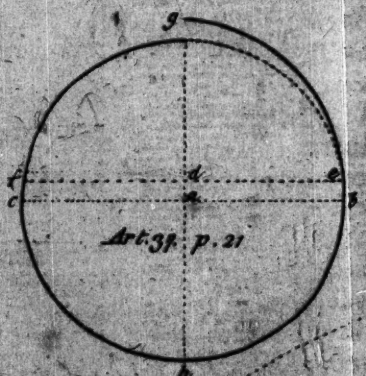
XX. *Of the Improvement of the fourth Method.*

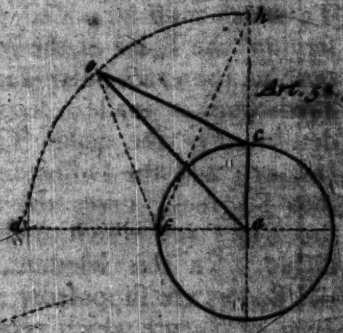
65. The Steps that have been taken for bringing Variation Charts to their preſent State, are as follows.

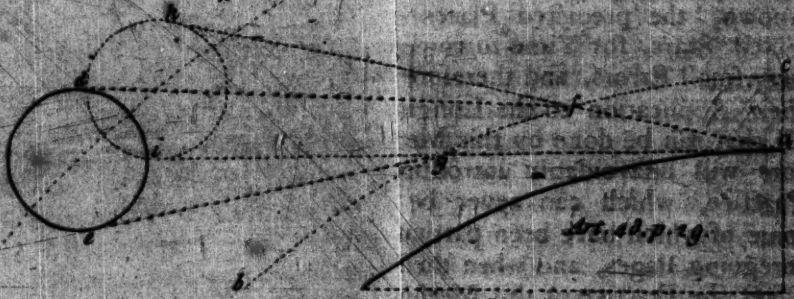
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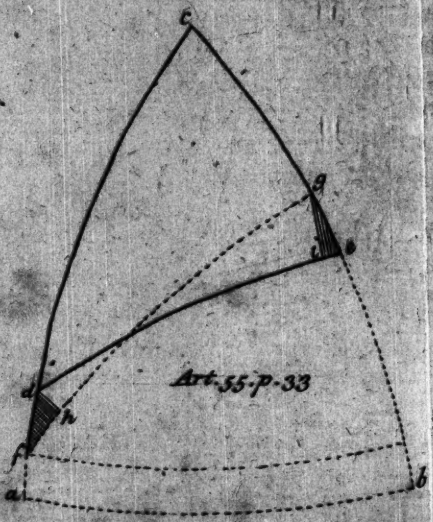
Plate 2

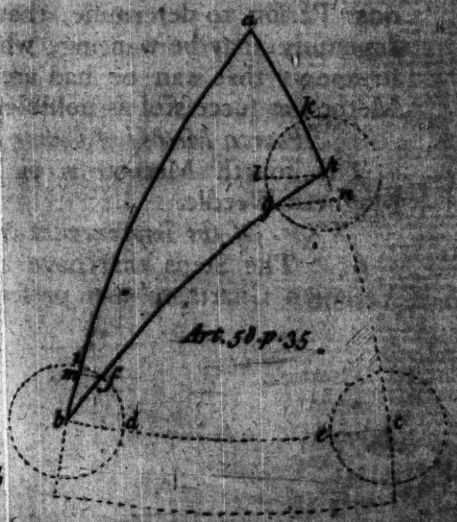
Refraction & Parallax.



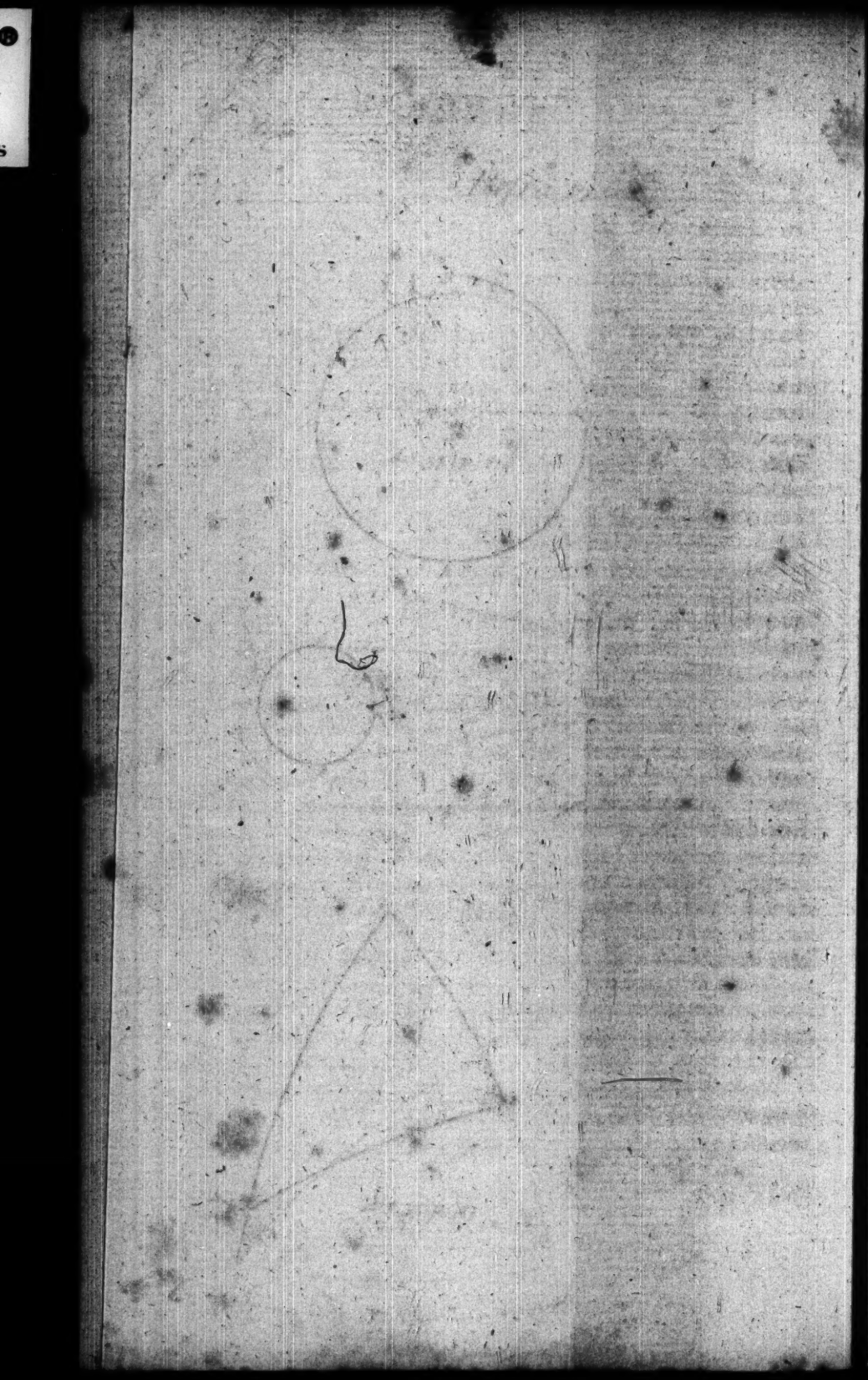








Published as the Act directs July 1. 1771 by S. Dunn.



THE INTRODUCTION PREFIXED TO THE NEW ATLAS OF VARIATIONS OF THE MAGNETIC NEEDLE.

By the AUTHOR.

1st. THE Charts and Delineations in this Work have been publiſhed at different Times, the Dates of which are reſpectively ſet to each of them. They are deſigned for aſſiſting the ingenious Navigator in determining the Longitude, whilſt he is croſſing thoſe Oceans and Seas for which the Charts are drawn; the Tendency of which is, to leſſen the Hazard which occurs in long Voyages, and to facilitate Commerce, for the univerſal Benefit of Mankind.

Perfection of Navigation.

2d. The Determination of the Longitude at Sea is all that is wanted to perfect the Art of Navigation, the other Parts being already brought to a great Degree of Exactneſs; but, when this latter Part is compleat, the Navigator will know, 1ſt, on what Courſe to ſail from one Place or Port to another, croſſing the Oceans and Seas whatever be their Extent, the Latitudes and Longitudes of the Places ſailed to and from being firſt ſettled and known; with all the Caſes incident to this Problem, ſuch as the Diſtance to be ſailed, the Departure from the Meridian, and other Things which now are called Navigation by the Log-line and Compaſs, independent of any Aſtronomical Obſervations. 2d. The Latitudes and Longitudes of Places, to and from which Ships ſail and near to which they paſs, will be known ſufficiently correct for the Uſe of Navigators. 3d. Whenever the Ship hath been diverted from her Courſe by Winds, Weather, or other Accidents, the Navigator will be able to determine the Longitude as well as Latitude of the Place where the Ship is, and conſequently the Courſe and Diſtance to be ſailed to the deſired Port. The firſt of theſe three Parts hath already been largely written of.—The ſecond Part may be accurately and eaſily compleated by the Method which I have delivered in a Treatiſe of *Practical Aſtronomy*, publiſhed near the End of the Year 1774.—And the third of theſe Parts may be greatly improved univerſally, and in many Places of the wide Oceans, which are far remote from Land, compleated, by the Method obſerved in this Work.

3d. In the Practice of Navigation, there are uſually reckoned five different Kinds of Sailing. 1ſt. Parallel Sailing. 2d. Plain Sailing. 3d. Middle Latitude Sailing. 4th. Great Circle Sailing. 5th. Mercator's, or rather Wright's Sailing. The firſt of theſe is ſtrictly true, but applicable only in Sailing coincident with a Parallel of Latitude, commonly called an Eaſt or Weſt Courſe. The ſecond is true only near the Equinoctial Line.

Invention of the Sea Chart.

Line. The third may be applied without much Error, when the Difference of Latitude and Difference of Longitude relative to the Queſtion comprehend no great Surface of the Earth's Globe; becauſe then it may be conſidered as being but a little different from a Plane Surface, and treated as ſuch. The fourth is applicable only in ſmall Diſtances, becauſe the Arch of a Great Circle on the Surface of the Sea doth not coincide with the Rhumb-line. The fifth is not only univerſally true, but eaſily applicable for all Latitudes and Diſtances from the Equinoctial to the Poles, and by Help of a Table of Meridional Parts calculated on the Principles of this Method, ſolves all thoſe Caſes of Sailing, either by the Conſtruction or Calculation of Plane Triangles, which would otherwiſe require a difficult Application of Curves or Spiral Lines.

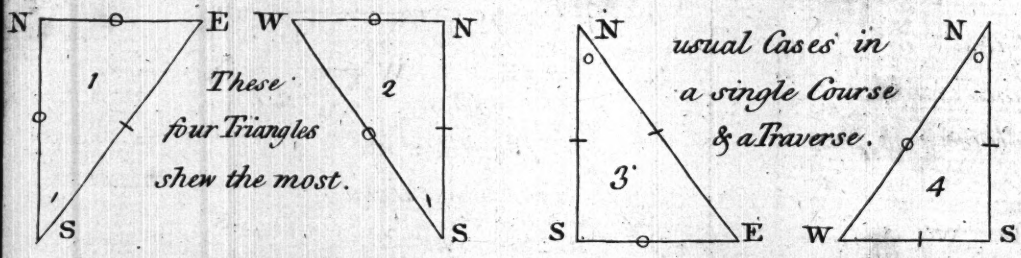
4th. The Invention of this moſt ingenious Conſtruction is commonly attributed to *Gerard Mercator*, a Native of Rupelmond in Flanders, who in the Year 1550 publiſhed a Map of the World, the Degrees of which were lengthened in approaching toward the Poles, and in 1554 he publiſhed a Map of Europe of the ſame Kind; after which, his Son publiſhed other Maps conſtructed on the like Principles; but it appears that thoſe ingenious Geographers rather aimed at a Correction of the Geographical Bearings of Places, than reducing the Rhumb-lines to Rectilinear ones; and that *Edward Wright*, who at the Age of 22 Years was admitted at Cambridge, and who in 1589 accompanied the Right Honourable the Earl of Cumberland in a Voyage to the Azores, was the original Inventor of this Chart, and that he publiſhed the Conſtruction of it in his Work, entitled, *Certain Errors in Navigation detected and corrected* in 1599. By which Work it alſo appears, that in 1594, 1595, 1596, and 1597, he, with great Ingenuity, obſerved at London the Solſtitial Altitudes of the Sun, and thereby determined the Obliquity of the Ecliptic 23° 28′, as it had been found 90 Years before

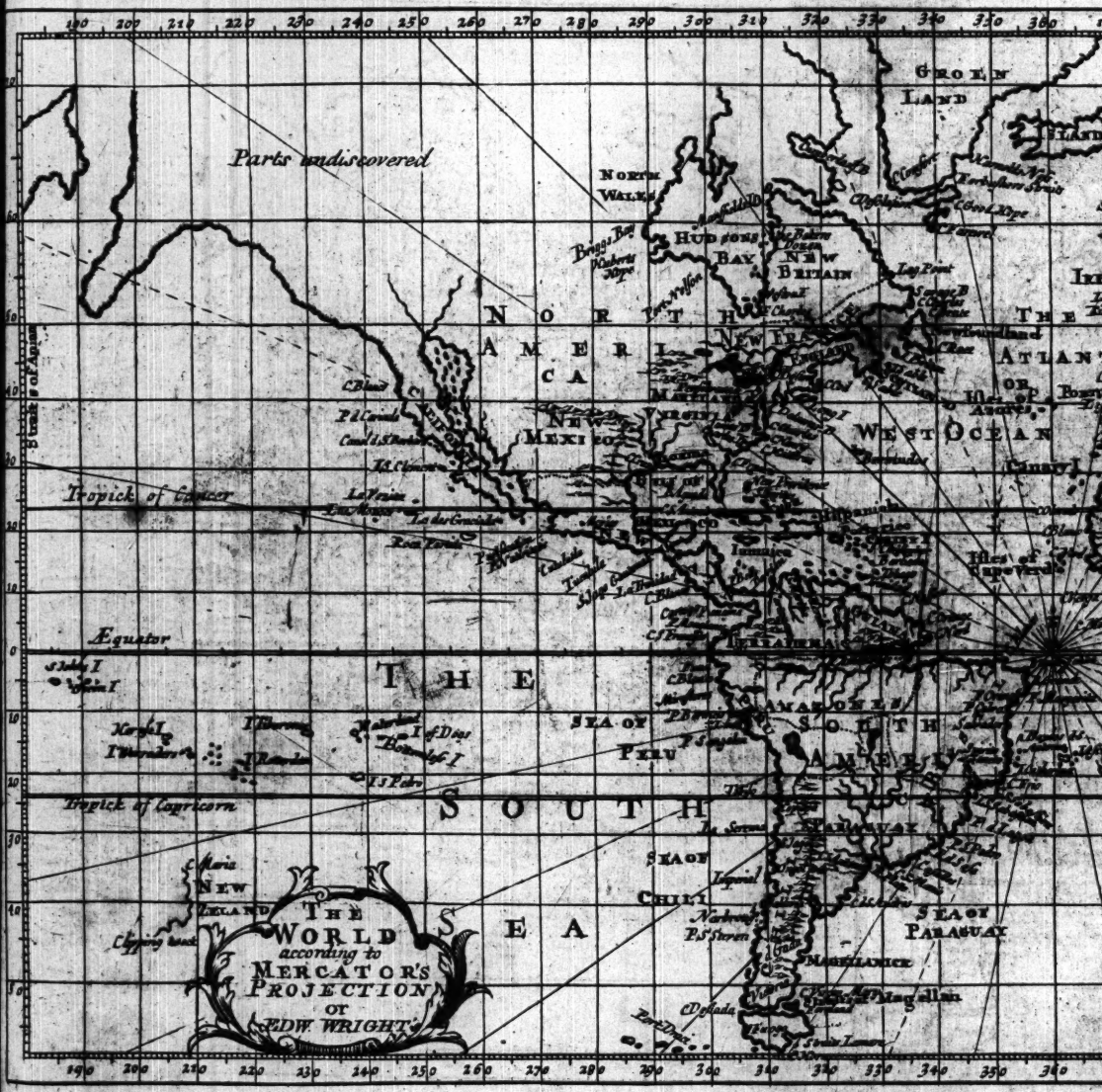
by the celebrated *Nicholas Copernicus*, Reviver of the true Syſtem of the World, whilſt they both conſidered the Effect of Refraction and the Sun's Parallax inconſiderable at great Altitudes; but Mr. *Wright*, leaving *Copernicus*, and adopting the Refractions and Parallax ſettled by the noble *Tycho Brahe*, which were not correct, thereby fell into an Error, and made the Obliquity of the Ecliptic 23° 31′ 30″, according to which he formed his Solar Tables. In 1616 he died, ſoon after his Return from viſiting Lord *Nepair* in Scotland, firſt Inventor of the Logarithms; and in 1618 his Son, *Samuel Wright*, publiſhed his Father's Tranſlation of the Logarithmic Treatiſe, which had been publiſhed by Lord *Nepair* in Latin, and dedicated it to the Right Honourable and Right Worſhipful Company of Merchants of London Trading to the Eaſt Indies, mentioning their Favours towards his deceaſed Father, their Employment of him in Mathematical Affairs, and the Utility of them in long and dangerous Voyages to and from the Eaſt Indies.

5th. Notwithſtanding the Utility of thoſe Improvements, there are five Kinds of Errors to which Navigation is ſtill ſubject, by the uſual Method of Sailing. 1ſt. The Diſtance meaſured by the Log-line may not be correct. 2d. It may not be practicable to keep the Ship in that Direction which is the true one, although it be known. 3d. There may not be true Allowance for Lee-way. 4th. There may not be due Knowledge of the Currents. 5th. There may be ſuch different Quantities of the Variation, during certain Intervals of Time, whilſt the Ship is ſailing, that nothing leſs than a daily Obſervation of the Variation would give it exact enough for Uſe; and ſuch Obſervations may not be had. The three firſt of theſe Errors may be corrected, ſo as to become leſs conſiderable, by Help of proper Experiments. The Errors of the fourth Kind muſt be found at the Times and Places when and where the Currents act; and the Errors of the fifth Kind may be wholly removed and annihilated by the Uſe of correct Variation

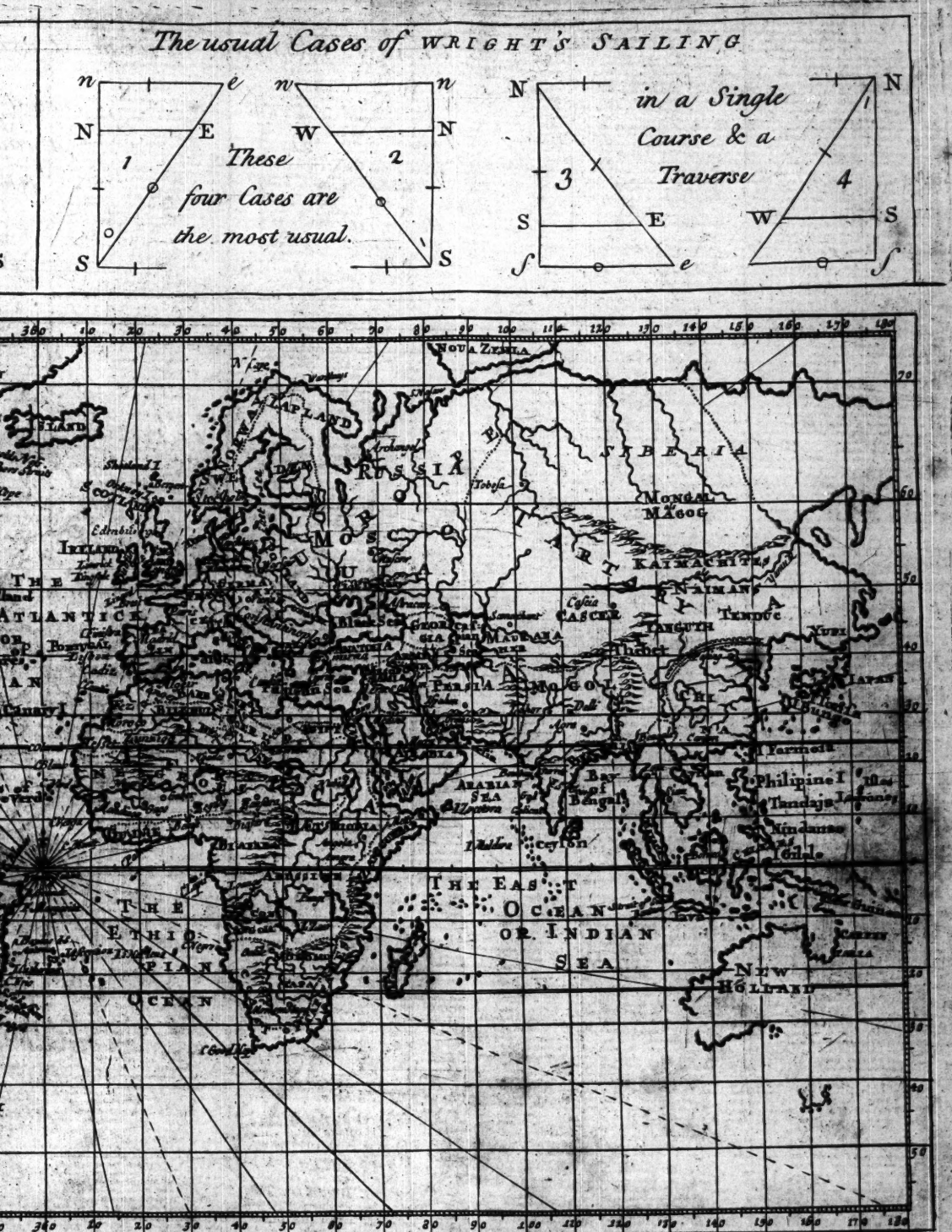
Charts

PLAIN SAILING in it's most usual Cases.



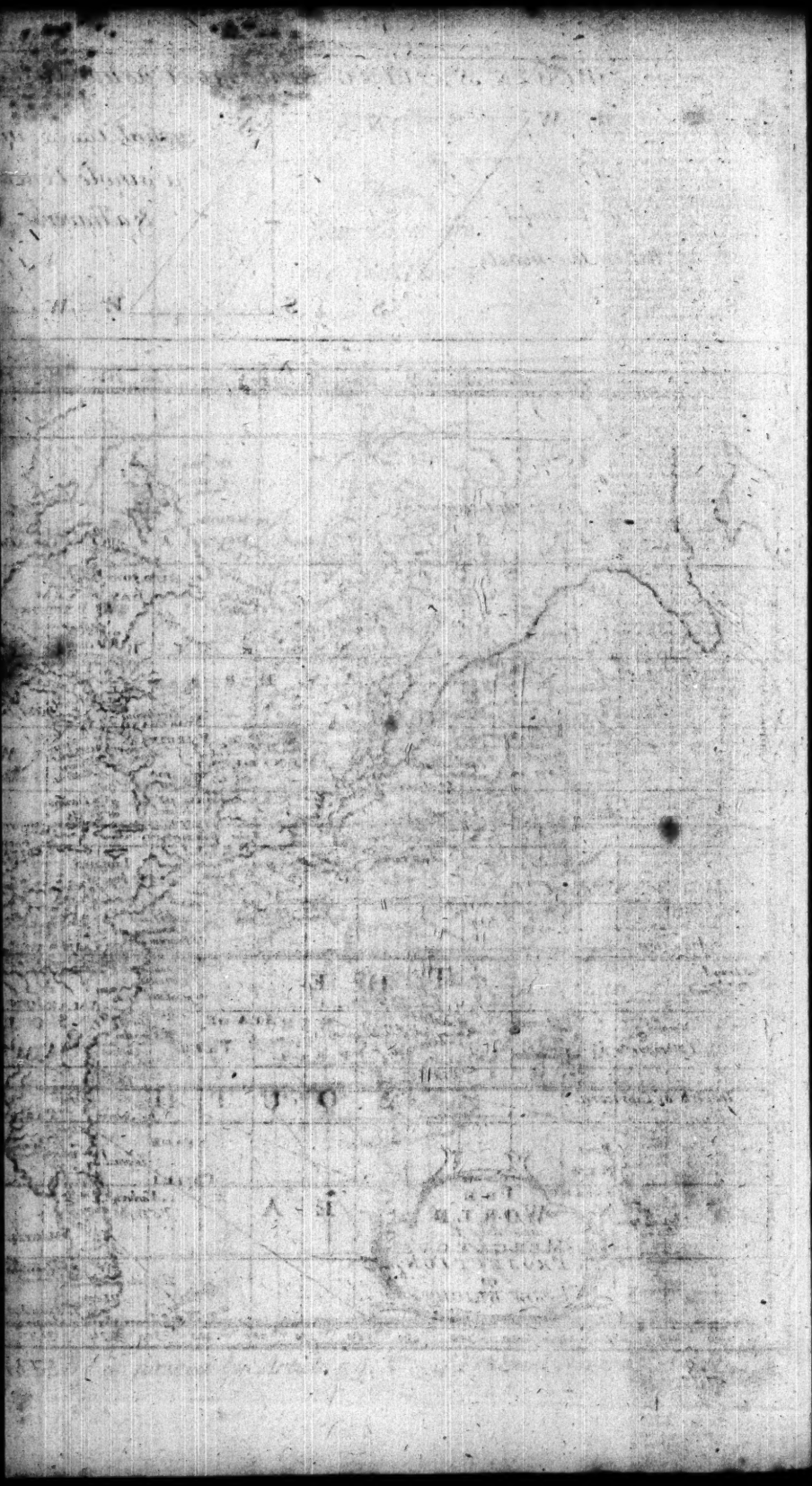


This Chart is formed



formed by Article 39. N°. 3. of Practical Navigation.

Plate 3.



Properties of the Sea Chart.

Charts of the Oceans which the Ship croſſeth. And therefore, if one of theſe five Errors be wholly annihilated, and three of them are ſuch as to have their Limits and Improvements; by the Application of correct Variation Charts, a fifth Part of the Errors which happen in the Ship's Reckoning will be taken off, could they be reckoned to ariſe from Cauſes equally difficult to be aſcertained; but, as that is not the Caſe, a much greater Correction may thereby be made.

6th. The firſt good Effect ariſing from this Invention of the true Sea Chart is, that in it the Rhumb-line Bearings are ſtraight Lines, and conſequently, by Help of a ſtraight Rule and a Pair of Compaſſes, thoſe Bearings are eaſily ſhewn by the Chart. This is the Reaſon why the ſtraight Lines are drawn croſſing the Oceans in this Chart, which being drawn in the plain Chart frequently miſlead the Navigator when he followed the Courſe indicated by them, in ſailing to Places which were accurately laid down by their obſerved Latitudes and Longitudes. The ſecond good Effect is, that all the Caſes of Sailing are ſolved by Proportions, amongſt which are the following. As Radius, is to the Diſtance ſailed; ſo is the Sine of the Courſe, to the Departure.—And as the true Difference of Latitude, is to the Departure; ſo is the Meridional Difference of Latitude, to the Difference of Longitude.—Alſo, as Radius, is to the Tangent of the Courſe; ſo is the Meridional Difference of Latitude, to the Difference of Longitude.—Alſo, as in the plain Chart; as Radius, is to the Diſtance ſailed; ſo is the Coſine of the Courſe, to the Difference of Latitude.—And as the Coſine of the Courſe, is to the Difference of Latitude; ſo is the Sine of the Courſe, to the Departure. From which a Variety of other Proportions may be formed.

7th. Another Advantage which this Chart has; the Courſe may be accurately ſet off on it, and alſo the Diſtance ſailed, by obſerving a proper Method. The Number of ſtraight Lines, which are uſually drawn on the

the Chart, render them almoſt unintelligible and difficult to be applied, and after all, the Subdiviſions of the Points of the Compaſs muſt be gueſſed at. As a Remedy to this Inconvenience, I have ſubſtituted the Circle itſelf with its Points and Subdiviſions, and alſo with its Degrees, which may be applied in the ſame Manner as the former Lines, and have ſhewn by Experiments, in my Practical Aſtronomy, that no conſiderable Error can ariſe in this Practice.

8th, The Stability of the Poles of the Celeſtial Equator, and the almoſt Spherical Figure of the Earth, enable the Navigator to determine the Latitude at Sea to a great Degree of Accuracy, by the Application of angular Inſtruments applied vertically. If the Syſtem of the World were ſuch, that a Celeſtial Body apparently moved round and near the Equator, from Weſt to Eaſt, whilſt the Earth is moving round its Axis the ſame Way, there would be an apparent Separation from the Sun 360 Degrees in a Day, and the Longitude near the Equinoctial would thereby be as accurately found, as the Latitude may now be taken by Meridian Altitudes. But the Moon's Receſſion, even from the Fixed Stars, being nearly 14 Degrees in a Day, the Certainty with which the Longitude at Sea may be taken by the Moon's Receſſion, although it affords the largeſt Data for putting this Method into Practice, will be but little more than a 25th Part of that with which the Latitude may be taken; and therefore, if the Multitude of Circumſtances, which tend to embarraſs the Problem, could be got over, there would ſtill be as much Difficulty to find the Longitude at Sea by the Moon, without erring 50 Miles at the Equinoctial, as in finding the Latitude without erring 2 Miles. This is the principal Cauſe why it is ſo extremely difficult to take the Longitude at Sea by the Moon, and why a greater Degree of Accuracy may be more uncertain.

9th. The Variation of the Magnetic Needle, or its Horizontal Deflection from the true Meridian of the

Place of Obſervation, hath been conſidered as of the greateſt Importance in Navigation, by every Man of either Learning or Ingenuity in Mathematics, Philoſophy, or Nautical Affairs, who hath given his Opinion concerning it. Hypotheſes have been formed, but none of them have agreed with Obſervations, nor hath it been poſſible to draw the Variation-lines by them. The Time when I began to think of laying a Plan for the obtaining of perpetual and accurate Variation Charts was in 1769; then I publiſhed Propoſals for printing ſeveral intereſting Particulars relative to the Longitude, but could not have proper Aſſiſtance for executing it. In 1774 my Practical Aſtronomy was publiſhed, in which is a New Method of obtaining perpetual and accurate Variation Charts, by Help of correſpondent Obſervations on Lunar Eclipſes, by which that Buſineſs may be effected without the Help of any Theory of the Magnetic Syſtem. In 1775 I publiſhed Two Sheets, one of the Atlantic Ocean North of the Equinoctial, the other South of the Equinoctial, ſhewing the Variations for 1776. It may ſeem ſurprizing, but it is true, in going through the principal Parts and almoſt all thoſe vaſt Oceans, I wanted no more to begin and finiſh the Work by than Obſervations made of the Variation at three or four Places near the Lands, and the other Places on the Oceans became regulated by a Theory which I invented, theſe are the Words, in a Pamphlet publiſhed with thoſe Charts. By the Word Theory, I mean what indicates the Cauſe of the Variation, at different Places and Times, and plainly demonſtrates it by Mathematical Principles and Philoſophical Laws. The drawing of accurate Variation Charts of the Magnetic Needle by a Theory muſt ſound very ſtrangely to Philoſophers, as nothing of this Kind hath hitherto been thought poſſible; and that the Longitude at Sea may be determined by ſuch Charts, where the Variation-lines run near enough to each other, and not too much in an Eaſt or Weſt Direction, hath been confirmed by the Teſtimony of

able Mathematicians and Navigators. In delineating thoſe Charts, the Prejudices which I had to overcome were ſo great as to exceed almoſt all Belief; for Inſtance, near the North Coaſt of North America, where Dr. Halley had given 10° W. Variation for 1700, and other Charts 14° W. Variation for 1744, and 20° W. Variation for 1756; and which by the ſame Way of Reaſoning might have been ſuppoſed 26° W. in 1776; by my Theory it was at thoſe Places no more than a fourth Part of that Variation. Two or three Obſervations, in theſe extreme Parts, being applied with the Theory, would have prevented any Error in that firſt Northern Sheet, but no ſuch Obſervations could then be had, and inſtead thereof a bad Obſervation, uſed inſtead of a good one, biaſſed the Lines, and made them, in thoſe Parts, a little different from the Theory; as a Remedy to which, I have made that Chart anew, wholly from the Theory, and have drawn the Atlantic Ocean quite through, on the ſame Principles, to be compared with Obſervations.

10th. The Impartial will not expect to find a Thing of this Kind perfected at once. Sir Iſaac Newton's Diſcoveries concerning the Solar Syſtem, require many Obſervations of the niceſt Kind to render them of Uſe in Navigation. The Theory of Magnetiſm requires a far leſs Number, and the Variation Charts may be ſo improved, as to be of great Utility to the Navigator; if not, ſo as to furniſh him with all that he wants. In the other Methods of finding the Longitude at Sea, the Errors ariſe from latent Cauſes not eaſily to be predicted, nor always to be accounted for; in this Method, Errors in the Variation Charts may eaſily be corrected ſo as to undeceive the Navigator in all Parts where they are applicable for the Longitude, and where they are not, and from Miniature may be extended to any Scale whatſoever. Obſervations prove, and the Theory demonſtrates, that in the Midſt of thoſe vaſt Oceans the Data are ſufficient for the Longitude, and that the Syſtem

is

Progreſs of this Improvement.

is leſs diſturbed by Inequalities there, than near the Lands. The Diſcovery of a Method of fixing ten thouſand, or a much greater Number of Marks, on the Oceans, ſhewing the Longitude, would have deſerved no little Regard: yet this is a much greater Acquiſition, without Expence, and therefore is not only deſerving Attention but Encouragement, as there is Proviſion for it in the late Act concerning the Longitude at Sea.

11th. The Gentlemen who have granted me that Aſſiſtance which I wanted to bring this Work from its Beginning to the State in which it now appears, are thoſe in the Direction of the Honourable United Eaſt India Company for this Year, and the two preceding ones. The Gentlemen in the Direction this preſent Year are, *Richard Becher*, Eſq; *Benjamin Booth*, Eſq; *George Cuming*, Eſq; *William George Freeman*, Eſq; *Robert Gregory*, Eſq; *Richard Hall*, Eſq; *John Harriſon*, Eſq; *William James*, Eſq; *John Manſhip*, Eſq; *James Moffatt*, Eſq; *Samuel Peach*, Eſq; *Frederick Pigou*, Eſq; *John Roberts*, Eſq; *Thomas Bates Rous*, Eſq; *Thomas Rumbold*, Eſq; *Henry Savage*, Eſq; *Joſeph Sparkes*, Eſq; *John Stables*, Eſq; *John Smith*, Eſq; *George Tatem*, Eſq; *Edward Wheler*, Eſq; *Daniel Wier*, Eſq; *George Wombwell*, Eſq; *John Woodhouſe*, Eſq; the Gentlemen in the two preceding Years Direction, not in this Year's, were *Charles Boddam*, Eſq; *William Devaynes*, Eſq; *Henry Fletcher*, Eſq; *Peter Laſcelles*, Eſq; *John Michie*, Eſq; and *Nathaniel Smith*, Eſq;

12th. Nor am I under ſmall Obligations to ſeveral of the *Commanders* and *Officers* of the Eaſt India Ships, who have ſo readily communicated to me thoſe curious Obſervations which they have made at Places remote from Land, by which the Regularity of the Magnetic Syſtem has been indiſputably ſettled in ſo ſhort a Time, and which might have remained doubtful, had there been only the Ship's Reckonings to be uſed in their ſtead. Their Names I ſhall beg Leave to mention in the Treatiſe which deſcribes more particularly the Uſe of the Charts.

How two Charts to be uſed.

13th. In delineating the Coaſts in this Work, I have conſulted the beſt Authorities that could be had, notwithſtanding which, it may be ſuppoſed, that the Longitudes of many Places are not yet known within a Degree, for want of proper Obſervations. Such Errors will be very conſpicuous in Maps or Charts drawn to a large Scale, when they can be diſcovered, however accurate the Topographical Deſcriptions may be; for this Reaſon, I have not endeavoured to finiſh the Coaſts in the firſt Delineations, but have reſerved that to a future Publication and Improvement of the Work.

14th. Another Advantage which ariſeth from the Diſcovery of a Theory of the Variation is, that the Variation Charts may not only be drawn from a few Obſervations, but they may be drawn for Years paſt and to come; from which ariſeth, an eaſy Method of making them applicable for the intermediate Years, with Errors which are very inconſiderable. From this Conſideration, if a Variation Chart be drawn for the Year A, and another for the Year B, and C be the medium intermediate Year between A and B; whilſt they are not a great Number of Years from each other, the Medium of the Variations found by both the Charts A and B, will be the Variation for the Year C, and for all the other intermediate Years, with but little Difference from Truth, and therefore two ſuch Charts, being well drawn, may be uſed as one Chart, without much Error, for thoſe Years. This is another Advantage in the Plan of this Work, by which it will appear, when and where the Motion of the Variation is either direct, ſtationary, or retrograde, and ſuch other Circumſtances as may occur, whilſt it is under the Regulation of its own peculiar Laws. If the Magnetic Syſtem was ſuch, that there could be no Variation any where, no Errors could ariſe in Navigation for Want of knowing its Quantity, as hath been before obſerved. Where the Variation-lines continue for a long Time nearly the ſame, and are ſmall in Quantity, and at the ſame Time ſpread widely on the Ocean, nearly the ſame Advantages will be derived

Where Longitude found by Variation.

derived from them; this is their State at ſeveral Places near the Lands, where they will not give the Longitude. At the other Places where they are either ſtationary, direct, or retrograde, running nearly North and South, and at the ſame Time nearer to each other, the Longitude is determinable by them. This latter is their State throughout a great Part of the Atlantic Ocean, and particularly the more central Parts, in which Ships loſe their Longitude, and do not find that Error until they come near to the Shores. But they are advantageous for all the Ethiopic and Southern Ocean from the Coaſt of Africa Eaſtward, to the Coaſt of South America Weſtward. In the Indian Ocean, they are applicable for the Longitude from the South Part of Africa Eaſtward, and North-Eaſt from the Eaſt Coaſt of Africa to the Weſt Coaſt of India. In the other Parts Weſt of the India-Iſlands and New Holland, they are not applicable for the Longitude; and in the greateſt Part of the Places where they are thus adapted for the Longitude, Experience ſhews, it may be taken by them, in moſt Caſes, within a Degree or 60 Miles.

13th. From what hath been here delivered; 1. Variation Charts of the Magnetic Needle accurately drawn are uſeful in Navigation, becauſe the Errors which occur in the Ship's Reckoning may be leſſened by them. 2. Such accurate Variation Charts can never be drawn by the uſual Method of Sailing. 3. To ſuppoſe they can be drawn by the Longitude as found by obſerved Diſtances of Sun and Moon or Moon and Stars, is to ſuppoſe that Method perfect, whilſt it requires a perfect Knowledge of the Refractions alſo of the Lunar Theory, the niceſt Inſtruments and Obſervers. 4. To ſuppoſe they can be drawn by the Application of Time-keepers, is to ſuppoſe theſe will not err more at Sea than whilſt they have been trying on Land, and that they will keep Time at both Places without Error. 5. A Theory of the Variation is diſcovered, by which, and a few Obſervations, the Variation-lines may be drawn more accurate, than by a Multitude of Obſervations and any other

The Utility of Variation Charts.

Method whatſoever. 6. Variation Charts have been drawn and publiſhed by this Theory, and found to agree with Obſervations at Iſlands in the Oceans whoſe Longitudes have been well aſcertained. 7. They have been compared with the moſt accurate Obſervations that have been made at Sea by the other Methods, and have indicated Imperfections in thoſe Obſervations. 8. By the Theory, thoſe Charts may be drawn for Years to come, as well as for the preſent Time, and thereby be ready for Service at all Times, at an eaſy Expence. 9. The Law of the Variation is ſuch, that in thoſe vaſt Oceans, at remote Diſtances from the Continents, the Variation-lines determine the Longitude; in which Places the Knowledge of the Longitude is moſt wanted, the ſhorter Diſtances to and from Places being determinable by the Reckoning. 10. The Variation-lines determine the Longitude at many Places near the Coaſts and Iſlands which are frequented in the moſt important Navigations; but there are other Places where they do not; this is an Imperfection in the Method of finding the Longitude by the Variation; but at thoſe Places the Navigator cannot be deceived by an imperfect Knowledge of the Variation, if he has thoſe Charts. 11. The Opportunities of taking the Variation are as numerous as the Appearances of the Sun, a Circumſtance which is incident to no other Method but a Time-keeper. 12. The Variation may be taken when the Sun is near the Horizon with peculiar Advantages, which the Lunar Method by Sun and Moon or Moon and Stars will not admit of, without great Diſadvantages, through Uncertainty concerning the Horizontal Vapours; and the like may be ſaid for ſmall Altitudes, without loſing the Aſcendency in greater Altitudes to the Zenith. 13. There are no angelic Powers required in taking the Variation, nor any Thing more than what Navigators are accuſtomed to do, ſo that here is required no new Syſtem of Operations to learn, nor new Books or Inſtruments to be furniſhed with. 14. An Improvement of the Variation Charts is manifeſtly, and by the

The Utility of Variation Charts.

univerſal Conſent of Men of the beſt Senſe in theſe Affairs, of Utility in Navigation and to the Public. 15. By accurate Variation Charts it may be known when the Celeſtial Bodies are near the Meridian, from which the Latitude may be inferred without accurate Meridian Altitudes or a Watch to keep the elapſed Time. 16. The Variation may be taken at all Times when other Obſervations can be made. Obſervations of the Variation with Compaſſes made in the uſual Manner have, in many Caſes, already determined the Longitude within half a Degree or 30 Miles. 17. If the Variation-lines were as advantageouſly poſited at all Places, as at many Places remote from Land, the Longitude might be determined by them within Half a Degree or 30 Miles univerſally; but as that is not the Caſe, if a conſiderable Allowance be made for their Deficiency, they determine Longitude in many Places, and improve Navigation in others, and therefore come within the Encouragement offered by the late Act of Parliament.

16th. The Life of Man is ſhort, and Science is extenſive. Men of Penetration and Underſtanding have ſtudied one Age after another for a long Series of Time, ſo that real Improvements in Science are but ſeldom to be met with, although the preſent State of the World is ſuch, that they are much wanted. This is one, which, a few Years ſince, no one could have expected to ſee for many Years to come. Envy ſhould therefore be ſilent, whilſt good Men by encouraging it become truly honourable. The Deſign and Endeavour is no leſs than to preſerve the Lives of thoſe who are Adventurers on the Ocean, for the Good of many, and the Properties of others who may be of the greateſt Conſequence amongſt Mankind.

17th. Charts properly conſtructed to leſſer and greater Scales, after *Wright's* Manner, will be of Uſe to Navigators, in indicating the Courſe to be ſailed from one Place to another, whether Diſtances be great or ſmall. Variation Charts of the Magnetic Needle, accurately

and

and properly drawn, will be of Uſe in allowing for the Variation. Both of theſe being applied at Sea, will enable the Navigator to purſue his Voyage in cloudy Weather, or when no Aſtronomical Obſervations can be made, with that Certainty which otherwiſe he cannot expect. It is the Plan and Deſign of this Work to inſtitute ſuch, but to complete it will require ſome Time and much Judgement. Topographical Deſcriptions erroneouſly placed become Errors in Geography. Hydrographical Delineations erroneouſly ſituated are Imperfections in Navigation. Blank Charts of *Wright's* Kind are uſeful for the Inſertion of ſuch Iſlands, Capes, or other Places, as the ingenious Navigator finds to have corrected; hereby the Courſe and Diſtance to and from them to other Places will be had with Certainty. The Longitude of the Ship being found by the Variation Charts, the Latitude and Longitude of that Place may be readily found in any of the *Wright's* Charts, and thereby the Courſe and Diſtance to other Places readily known. To treat fully on this Part is to enter on the Subject of Navigation, to conſider what are the proper Obſervations and Calculations, and what are proper Books to be uſed as Tables.

18th. In the intermediate Time, I wiſh to remind the ingenious Navigator of the great Utility correſpondent Obſervations of Lunar Eclipſes will be of, if obſerved and compared with each other, as recommended in my *Practical Aſtronomy*. Aſtronomers have never before thought it poſſible to find the Longitude of a Place, either at Sea or on Land, to that Exactneſs which I have there mentioned; and the Uſe of ſuch Obſervations is obvious. The remarkable Lunar Eclipſe of July 30th laſt was obſerved by me, with all deſirable Accuracy; and it will ſoon appear before the Public.

LONDON, October 28th, 1776. Samuel Dunn.

To

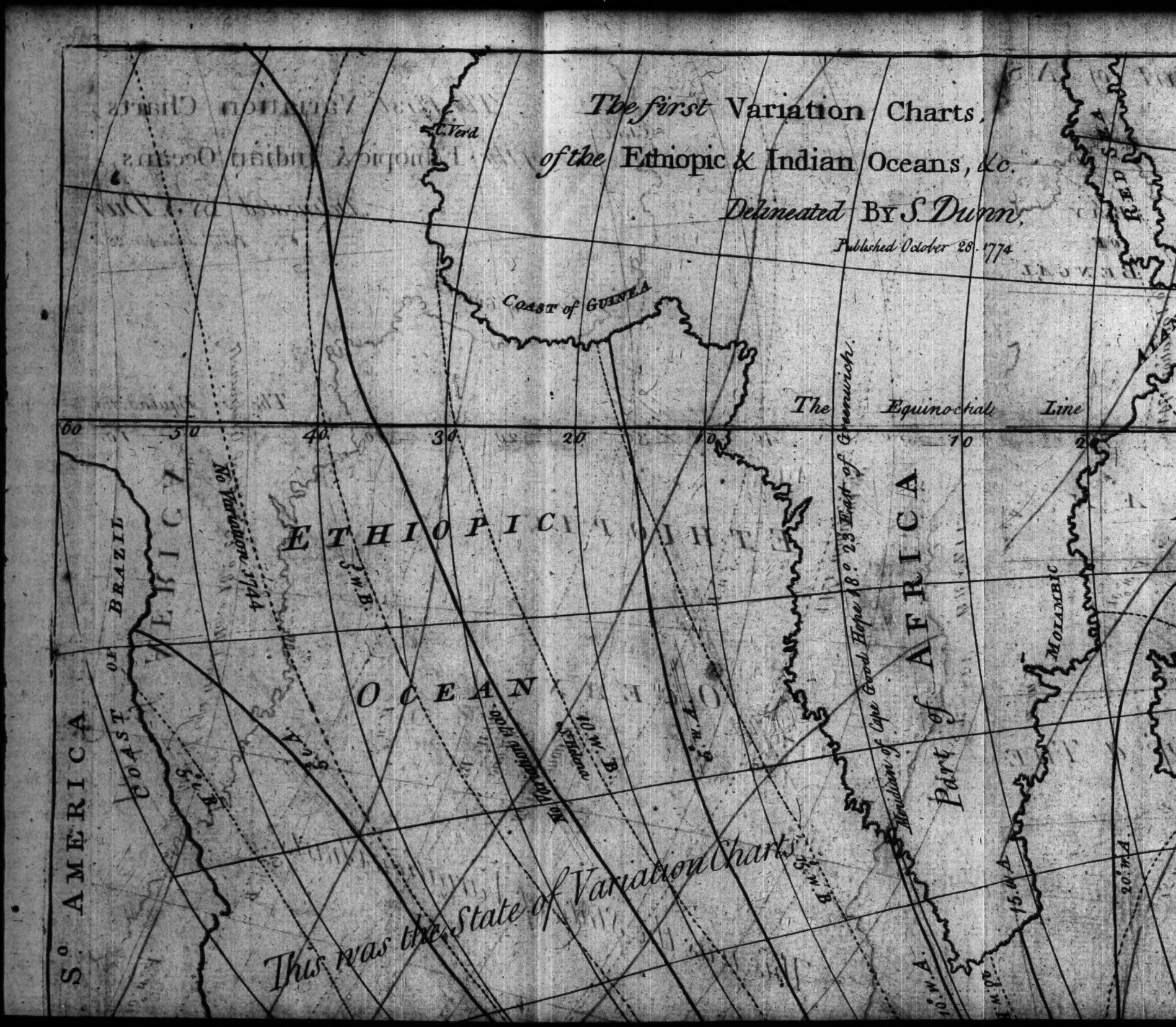
Of the Obſervation and Calculation.

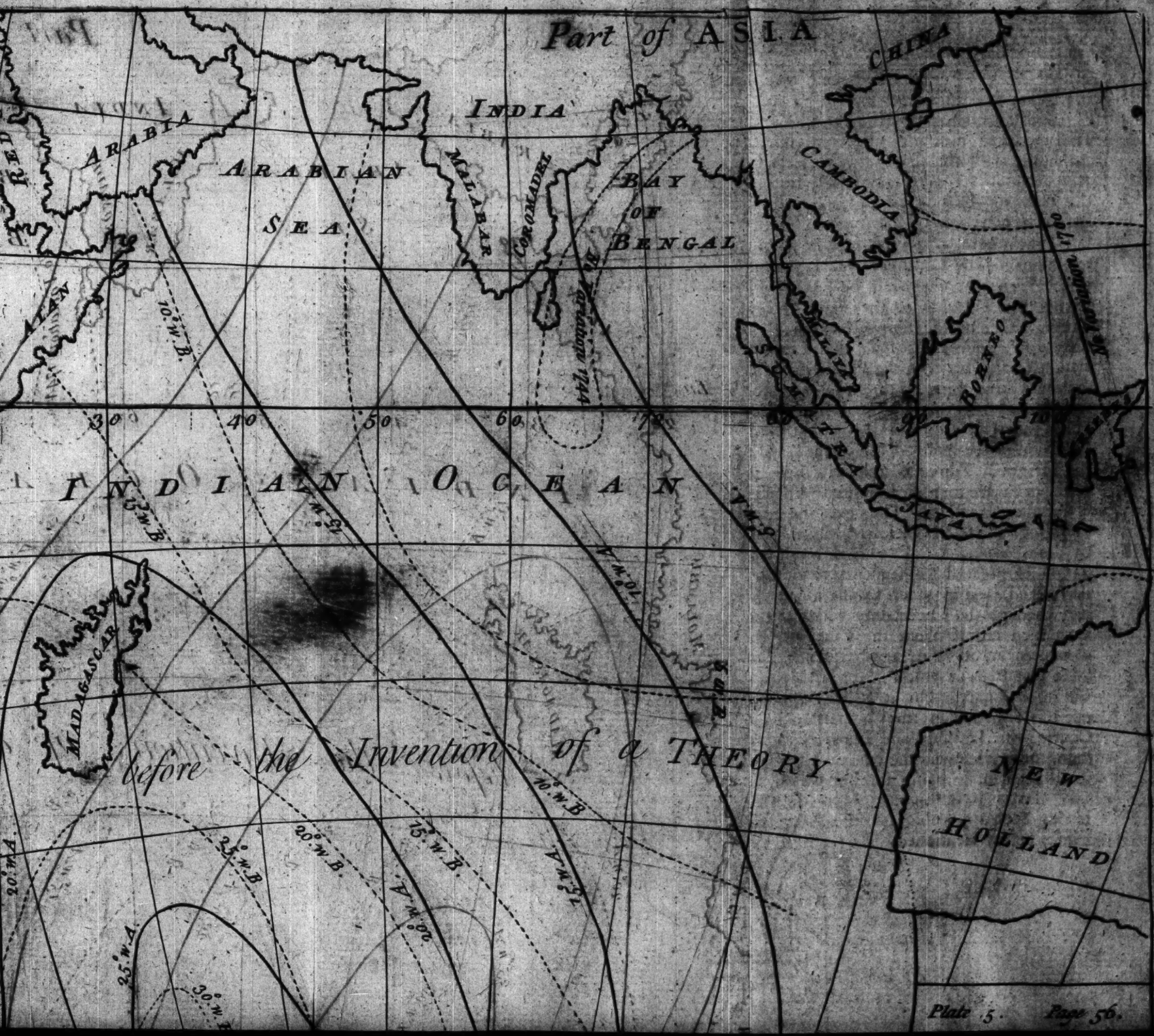
To which might have been added from the Pamphlet with my former Charts, the Method of calculating the Variation, the Inſtruments uſed, and the Application of the Charts, as follows:

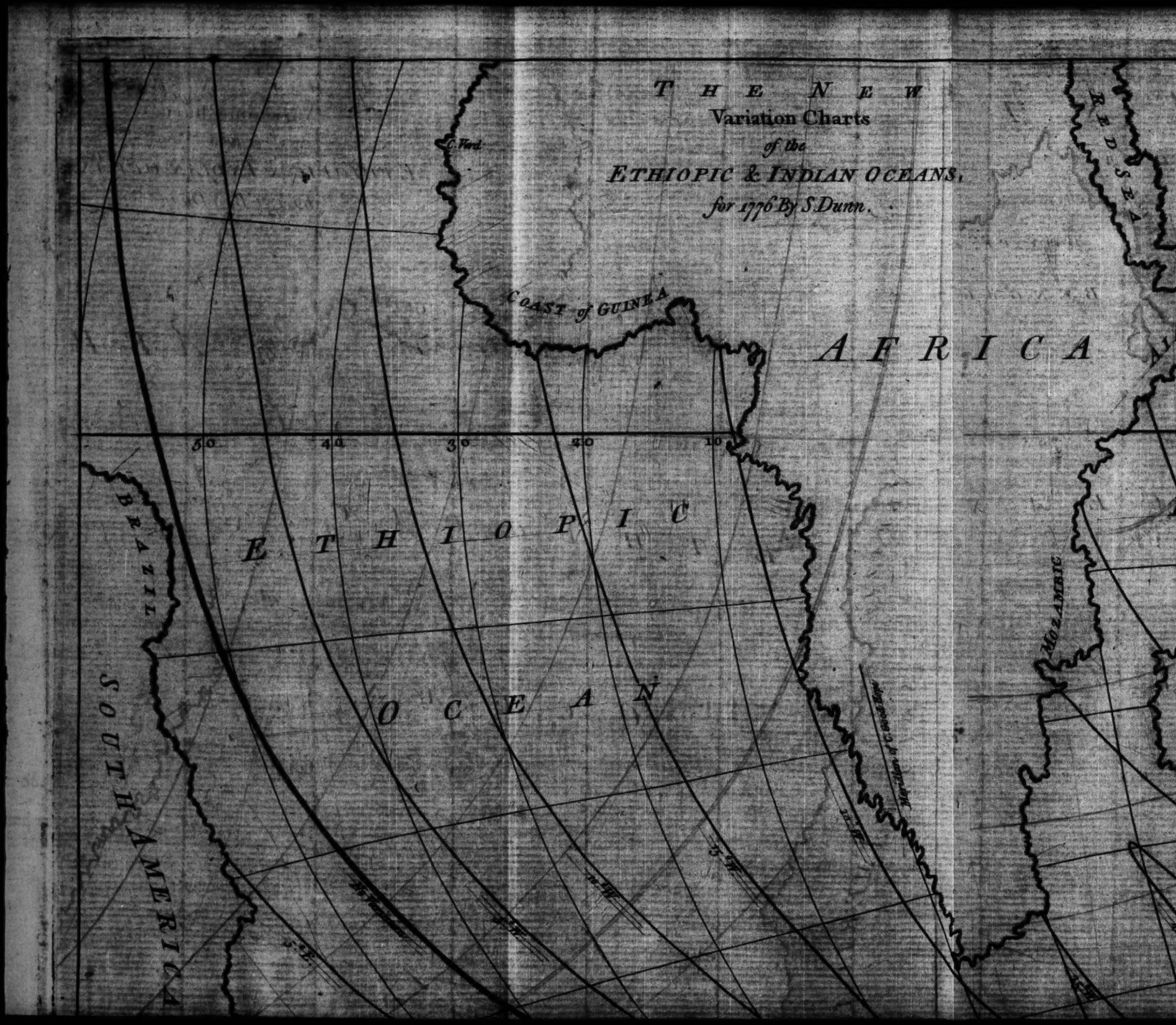
19th. The Inſtruments to be uſed are either an Hadley's Quadrant or Sextant. The Navigator ſhould likewiſe have the Uſe of an Azimuth Compaſs, which is to be applied for determining on what Point of the Compaſs, in Degrees and Minutes, the Sun bears at the making of an Obſervation. The Quadrant is for taking the Latitude of the Place, and alſo the Altitude of the Sun when the Longitude is obſerved for; no great Nicety is required in either of theſe, nor that the Azimuth Compaſs ſhould determine the Magnetic Bearing of the Sun nearer than to a Quarter of a Degree. A much leſs Accuracy may be diſpenſed with, if ſeveral Obſervations be taken nearly at equal Intervals of Time, and the Medium of them ſet down for the Obſervation. He ſhould likewiſe have a Table of Logarithmic Sines, Tangents, and Secants, ſuch are in the common Books of Navigation; the Variation Charts here deſcribed; a Table of the Sun's Declination; and, if he chooſes to obſerve for the Longitude by this Method in the Night-time, he ſhould have a Table of the Declinations of the principal Fixed Stars. The Proportion for the Amplitude is thus: As the Coſine of the Latitude is to the Sine of the Declination, ſo is Radius to the Sine of the Amplitude. But when the Azimuth of the Sun is required, a larger Proceſs is uſed as follows: The Co-latitude, the Co-altitude, and the Polar Diſtance, are written under each other, and the Half Sum of them taken, from which the Polar Diſtance being ſubtracted, leaves a Remainder; then the Log-cofecant of the Co-latitude, taking ten from the Index; the Log-cofecant of the Co-altitude taking ten from the Index; the Sine of the Half Sum, and the Sine of the Remainder, being added together, and their Half Sum found in Log-ſines, double the Arch to which that is a Co-ſine, is the Azimuth or True Bearing

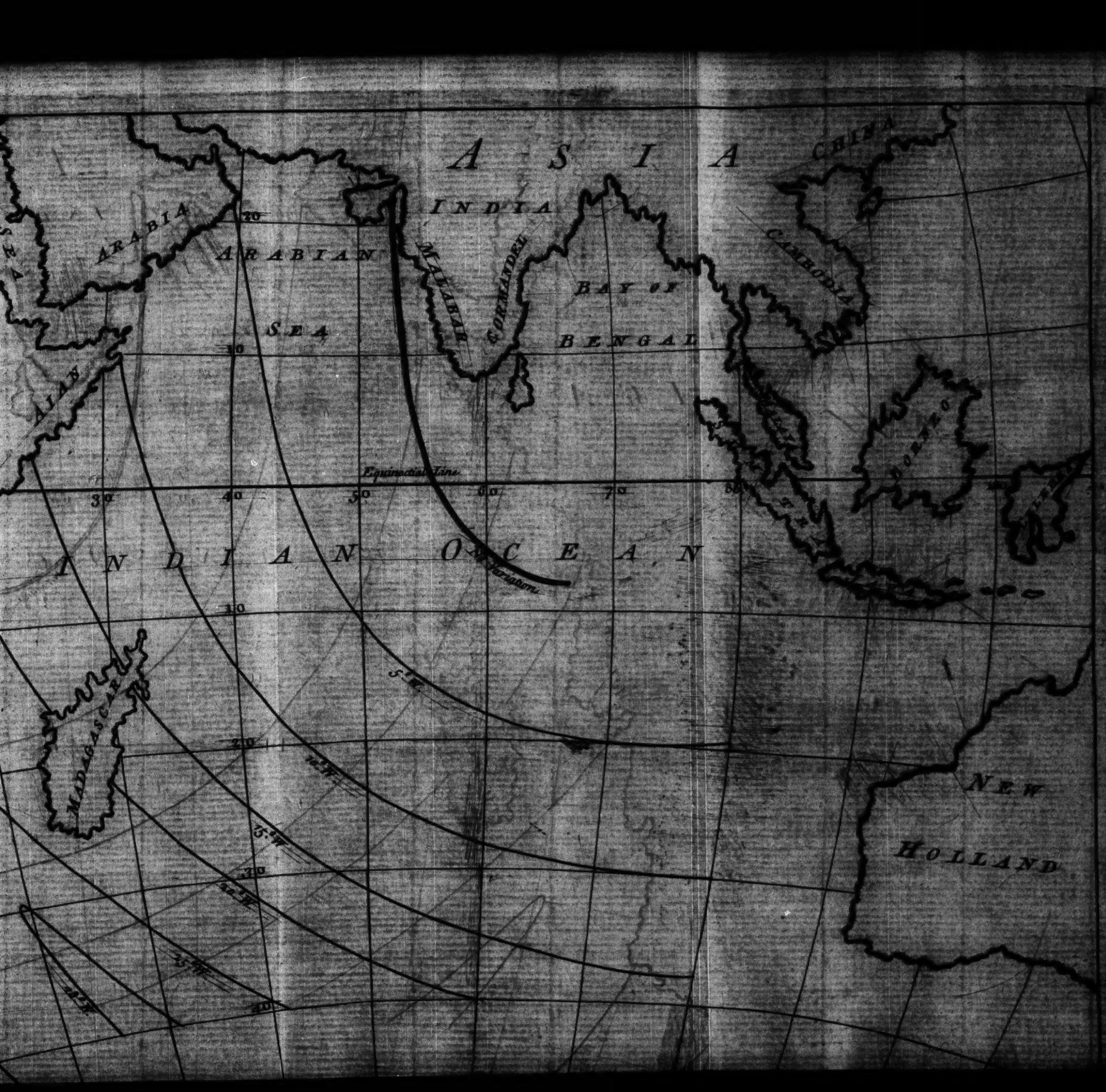
ing of the Sun from the Point of the Horizon under the elevated Pole, whether it be North or South Latitude. Having found either the true Amplitude or the true Azimuth, the Difference between it and the Magnetic Amplitude or Azimuth reſpectively is the Variation of the Compaſs Eaſting or Weſting, which are ſo well known to Navigators, that it ſeems needleſs to particularize them here. The like may be ſaid concerning Obſervations for the Latitude. Having found the Quantity of the Variation, whether it be Eaſting or Weſting, and having the Latitude of the Place ſomewhat near the Truth, either by the Ship's Reckoning or an Obſervation, guide your Eye along that Latitude, on the Variation Chart, between the Parallels, until you come to ſuch a Curve Line of the Variation as you have obſerved, and from that Place of Interſection or Meeting, guide your Eye Northwardly or Southwardly, under the Direction of the inclined Meridians; and the Figures in the Margin at Top or Bottom will ſhew the Degrees and Parts of a Degree of Longitude from London, either Eaſting or Weſting. Thus may the grand Problem of the Longitude at Sea be ſolved within the Oceans limited by the Charts, a vaſt Extent, through which the Navigation of greateſt Conſequence to the Britiſh Nation is performed, and in which by the uſual Method of Sailing, through the Uncertainty which attends it, and the Deception ariſing from the Currents, it is not to be wondered at, if Ships are frequently deceived thereby ſome Hundreds of Miles. Nor is this all the Uſe of the Variation Chart; but it is daily applicable for taking out the Variation of the Compaſs to be allowed in the Courſe, the Latitude and Longitude by the Reckoning being nearly known, the general Uſe of which is known to all Seamen, as being that for which all other Variation Charts have hitherto been drawn, without ever having been applicable for the Longitude, within the Limits before-mentioned.

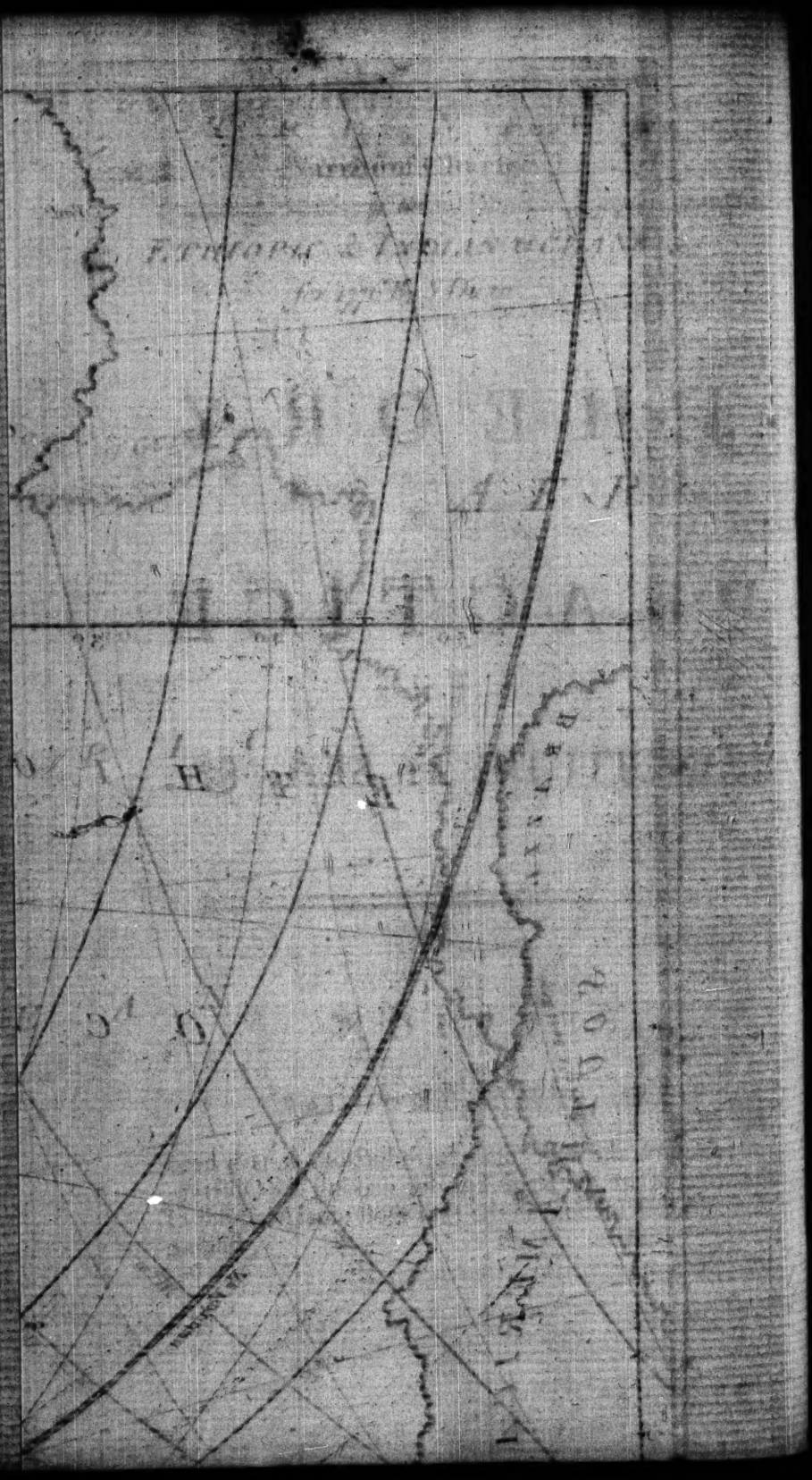
The


The first Variation Charts,
of the Ethiopic & Indian Oceans, &c.
Delineated By S. Dunn,
Published October 28. 1774
C. Verd
Coast of Guinea
The Equinoctial Line
60
50
40
30
20
10
0
10
20
No Variation 1744
S. W. B.
ETHIOPIC OCEAN
S. America
Coast of Brazil
Meridian of Cape Good Hope 18° 23' East of Greenwich.
Part of Africa
Mozambic
Red Sea
This was the State of Variation Charts


Part of ASIA
INDIA
CHINA
ARABIA
ARABIAN SEA
BAY OF BENGAL
MALABAR
COROMANDEL
CAMBODIA
BORNEO
INDIAN OCEAN
MADAGASCAR
NEW HOLLAND
JAVA
SUMATRA
Before the Invention of a THEORY.
Plate 5. Page 56.


THE NEW
Variation Charts
of the
ETHIOPIC & INDIAN OCEANS,
for 1776 By S. Dunn.
C. Verd.
COAST of GUINEA
AFRICA
RED SEA
50
40
30
20
10
ETHIOPIC
OCEAN
BRAZIL
SOUTH AMERICA
MOZAMBIC


A S I A
INDIA
CHINA
ARABIA
ARABIAN SEA
MALABAR
CORMANDEL
BAY OF BENGAL
CAMBODIA
BORNEO
Equinoctial Line
INDIAN OCEAN
MADAGASCAR
NEW HOLLAND
10
20
30
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70
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THE

THEORY

AND

PRACTICE

OF THE

LONGITUDE AT SEA, &c.

SECTION XXI.

Of the Longitude of the Coast.

Art. 66. IN the foregoing Introduction, it hath been shewn what are the Requisites for making the Observation and Calculation, in the four most usual Methods of finding

finding the Longitude at Sea; alſo the Accuracy to which it may be taken with the beſt Inſtruments and by the moſt expert Obſervers. It is neceſſary that the Nautical Obſerver ſhould be acquainted with theſe Things, that he be not deceived into an Expectancy of coming nearer the Truth, than the many Difficulties which are to be overcome in the Theory and Practice will permit, it being certain, that each of the Methods hath it's own peculiar Imperfection, and that the greateſt Attention and Care that can be taken, are not wholly ſufficient for removing them. Under ſuch an Exertion of the humane Faculties, Obſervers will wiſely conclude, that Navigation may ſometimes receive Aſſiſtance from one Method and at other times by another. That the Longitude found by the uſual method of Sailing is not always to be rejected becauſe it frequently becomes erroneous in long Voyages; but on the contrary, it is, for ſhort Voyages, the moſt general, and perhaps moſt perfect Method that ever will be diſcovered. That the Method by a Time Keeper is good, but not ſo univerſal as is deſired, ſince there are many ſuch Machines that are imperfect, to one that is perfect; and on the Contrary, a pretty good Watch may be of very great Uſe at Sea, beſides that of keeping correct Time during a long Voyage. That in the Lunar Method, the Obſerver is under the Neceſſity of entering into the minute Circumſtances, amongſt which, for want of finer Diſcernment, the Mind concludes with Uncertainty; but at the ſame Time, that this Method is founded on the Baſis of Nature, and is void of many Imperfections which are to be expected from many other Contrivances. That the Method by the Magnetic Variation ought not to be rejected whenever it can be applied; ſeeing it is, by far the eaſieſt of all the Methods that have been invented; and beſides, is the greateſt Addition to the uſual Method of Sailing, and the Longitude to be deduced therefrom, that can be named; and although there be ſome Seas where it cannot be practiſed, over thoſe ſame Seas frequently the

Lunar

Of the the Longitude of the Coaſt.

Lunar Method may not be practicable for want of ſeeing the proper Luminaries, whilſt over many other Seas the Lunar Method cannot be practiſed for the ſame Reaſon, but the Magnetic Method may, by only a ſingle Sight of the Sun.

67th. But before either of the four Methods can be practiſed at Sea with deſirable Succeſs, the Longitudes of Places near the Sea Coaſt (and particularly of Capes and other Points of Land which project toward the Ocean) ſhould be known, to ſuch Exactneſs as Navigation requires; becauſe, without this moſt neceſſary Preparation, none of the Methods of finding the Longitude at Sea, can become of general Uſe, although this doth not belong to the Longitude at Sea itſelf. For Inſtance, if a Ship ſails from London to the Lizard, and from thence to Madeira; but takes her Departure from the Lizard by the firſt or ſecond Method, the Longitude of the Lizard muſt be known before that of Madeira can be inferred by either of thoſe Methods. But by the third and fourth Methods there will be no Want of the Lizard's Longitude to infer that of Madeira; and the like holds for other Places, ſo that in this, the Variation Chart is as univerſal as the Lunar Method, and the only Difference is, how much one of them may be more or leſs accurate than the other. Otherwiſe, if the Longitude of the Coaſt be ſettled by the ſecond and third Methods together, and the Longitude found by the going of a Time Keeper, and obſerved Diſtances of the Sun and Moon or Moon and Stars be noted to agree with each other, there is no abſolute Certainty that the Watch hath kept to equal Time, nor that the Lunar Method is more exact than the Imperfection attending it may make either erroneous or doubtful; and therefore, this may want ſuch other Correction as is to be had from Obſervations on Land, by Methods more perfect than thoſe applicable at Sea.

68th. Of theſe Methods, their Perfection or Imperfection, I have given ample Account in my Treatiſe on *Practical Aſtronomy*, where the Reader may ſee the ea-

ſieſt and moſt compleat Methods whereby theſe Difficulties can be overcome, and with the feweſt or moſt portable Inſtruments; amongſt which Methods, having in a particular Manner recommended that by correſpondent Obſervations of the touch of the Penumbra to the Lunar Spots, in Lunar Eclipſes that are total, I ſhall beg leave to introduce here the Particulars of the Lunar Eclipſe mentioned, Page 54, as follows.

XXII. *Of the Lunar Eclipſe, July* 30, 1776.

69th. Having drawn a Meridian Line and adjuſted thereto the Apparatus deſcribed in my *Practical Aſtronomy*; being alſo provided with two Regulators which beat each Seconds of time, and another which beat Half-ſeconds of Time, and having brought all three to keep together for a conſiderable Interval, and ſet each ſomewhat near to mean Solar Time, without being preciſe concerning the Quantity by which they were too ſoon or too late, as that would be determined by Meridian Tranſits, I made the following original Obſervations,

Near Covent Garden, London, July 30, at Noon.

At 12h. 2m. 13s. per Clock; the Sun's Limb touched the Cylinder; at 12h. 8m. 36s. the other Limb left it; ſo the Middle was at 12h. 5m. 25s.

At 12h. 3m. 38s. the Sun's Limb touched the Thread; at 12h. 7m. 13s. the other Limb left it; ſo the Middle was 12h. 5m. 26s.

At 12h. 5m. 26s. the Threads appeared biſected.

The Mean Solar Time was before Solar Time, 5m, 57s. The Place of Obſervation 28 Seconds of Time weſt of Greenwich, and 6 Seconds weſt of St. Paul's, London.

Hence the Clocks were 32 Seconds after mean Solar Time; but 5m. 25s. before Solar Time for the Place of Obſervation; and 5m. 53s. before Solar Time for Greenwich, which being ſubtracted from any of the following Obſervations made concerning the Beginning, End, or touch of any of the Spots, will give, very nearly the Solar Time for Greenwich.

Having

Of the Lunar Eclipſe, July 30. 1776.

Having prepared the Clocks, I conſidered what Method was moſt proper for obſerving the Beginning and End of the Eclipſe, on Account of the Penumbra, and determined to obſerve theſe Parts, with the naked Eye, alſo with a Refractor magnifiing 15 Times, and with a Reflector magnifiing not leſs than 60 Times, but the Eclipſe of the Spots with the Reflector only.

Before the Eclipſe began, Mr. Eardley Norton, a very ingenious Clock and Watch Maker, and another Perſon, were ſo kind as to come and offer me Aſſiſtance, and brought two curious Watches, but I endeavoured to keep as cloſe as poſſible to the Time ſhewn by the Clock, beating Half-ſeconds of Time. With their Aſſiſtance in ſpeaking the Beats, I eaſily wrote down theſe following original Obſervations, as the Time was ſhewn by the Clock.

	Time per Clock h.	m.	s.
Beginning by the naked Eye	10	15	42
by Refractor 15 Power	10	16	25
by Reflector 60 Power	10	17	40
Kepler touched by the Shade	10	23	30
biſected	10	24	30
covered	10	26	—
Hereclides biſected	10	28	5
Copernicus touched by the Shade	10	33	—
biſected	10	34	33
covered	10	35	32
Plato touched by the Shade	10	42	15
biſected	10	42	48
covered	10	43	15
Tycho touched by the Shade	10	44	—
biſected	10	44	50
covered	10	55	42
Manilius touched by the Shade	10	48	5
biſected	10	48	46
covered	10	49	33
Mare criſium touched by the Shade	11	5	10
biſected	11	7	30
covered	11	9	5

Beginning

Of the Lunar Eclipſe, July 30. 1776.

	h	m	s
Beginning of total Darkneſs	11	13	26
End of total Darkneſs	12	48	55
Grimaldus begins to appear	12	52	—
biſected	12	52	30
uncovered	12	53	—
Kepler begins to appear	13	3	10
biſected	13	3	40
uncovered	13	4	15
Tycho begins to appear	13	5	35
biſected	13	6	15
uncovered	13	7	—
Hereclides biſected	13	7	—
Copernicus begins to appear	13	10	52
biſected	13	12	—
uncovered	13	13	—
Plato begins to appear	13	20	15
biſected	13	21	10
uncovered	13	21	35
Manilius begins to appear	13	26	10
biſected	13	26	35
uncovered	13	27	5
Pliny begins to appear	13	29	12
biſected	13	29	45
uncovered	13	30	15
Mare criſium begins to appear	13	41	20
biſected	13	43	40
uncovered quite	13	44	—
End by Reflector of 60 Power	13	46	55
Refractor of 15 Power	13	48	20
Naked Eye	13	49	—

The next Day, July 31. at Noon, the Sun's Meridian Tranſit was taken for finding the Error in the going of the Clock, by the ſame Method, and found thus.

At 12h. 2m. 37s. the Sun's Limb touched the Cylinder; at 12h. 8m. 24s. the other Limb left it; ſo the Middle was at 12h. 5m. 30s.

At 12h. 5m. 32s. the Threads appeared biſected.

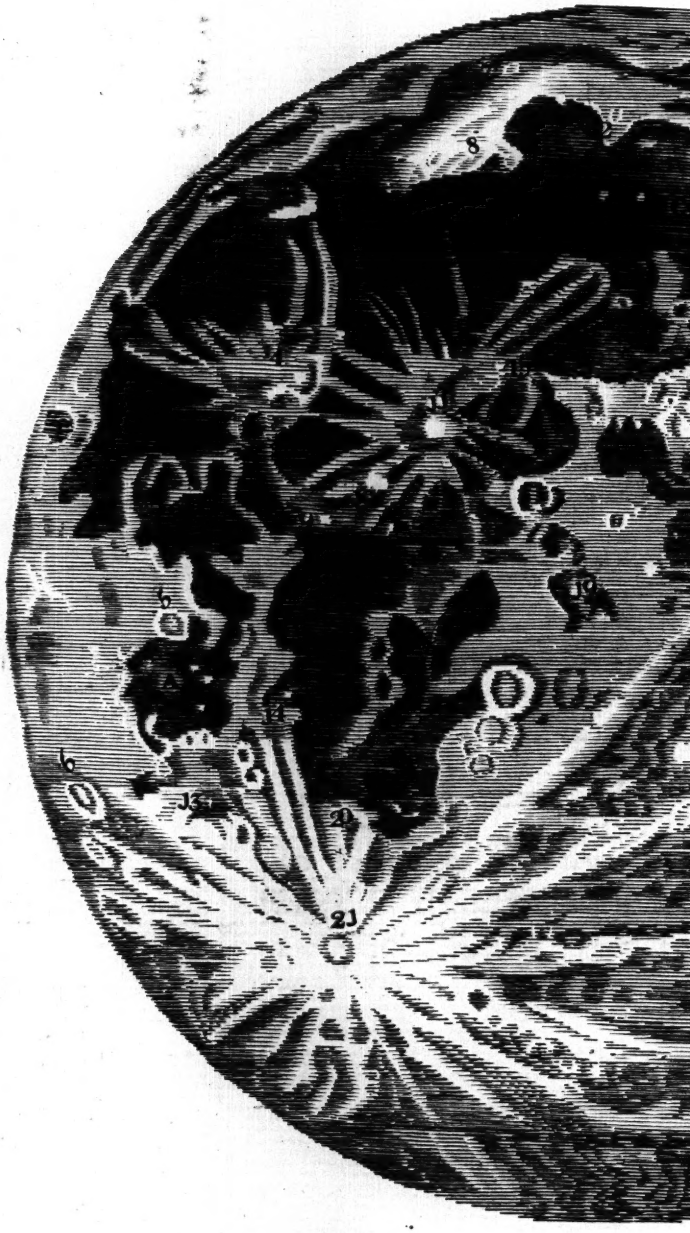
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The Names of the LUNAR SPOTS.

1	Grimaldus	25	Menelaus
2	Galileus	26	Hermes
3	Aristarchus	27	Possidonius
4	Kepler	28	Dionysius
5	Gassendus	29	Pliny
6	Skikardus	30	Theophilus
7	Harpalus	31	Fracastorius
8	Heraclides	32	Promont.m acutum
9	Lansbergius	33	Messala
10	Reinoldus	34	Promont.m somnii
11	Copernicus	35	Proclus
12	Helicon	36	Cleomedes
13	Capuanus	37	Snellius
14	Bullialdus	38	Petavius
15	Eratosthenes	39	Langrenus
16	Timocharis	40	Taruntius
17	Plato	A	Mare humorum
18	Archimedes	B	Mare nubium
19	Ins.a sinus medii	C	Mare imbrium
20	Pitatus	D	Mare nectaris
21	Tycho	E	Mare tranquillitatis
22	Eudoxus	F	Mare serenitatis
23	Aristotle	G	Mare fœcunditatis
24	Manilius	H	Mare crisium

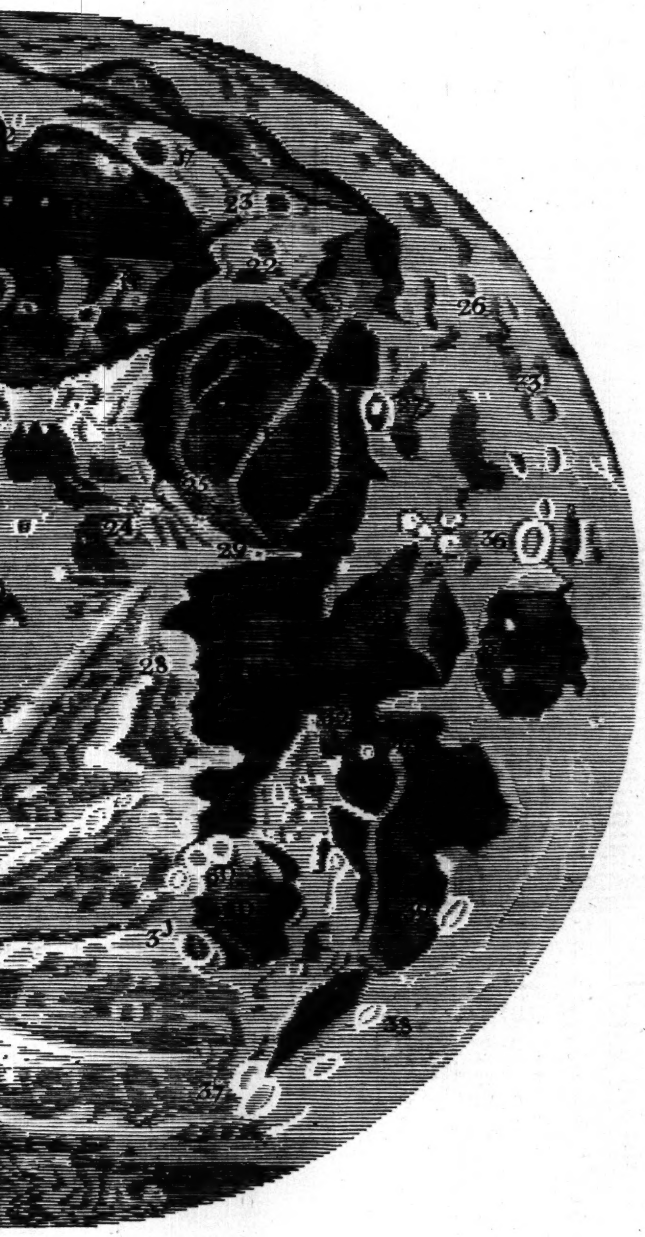
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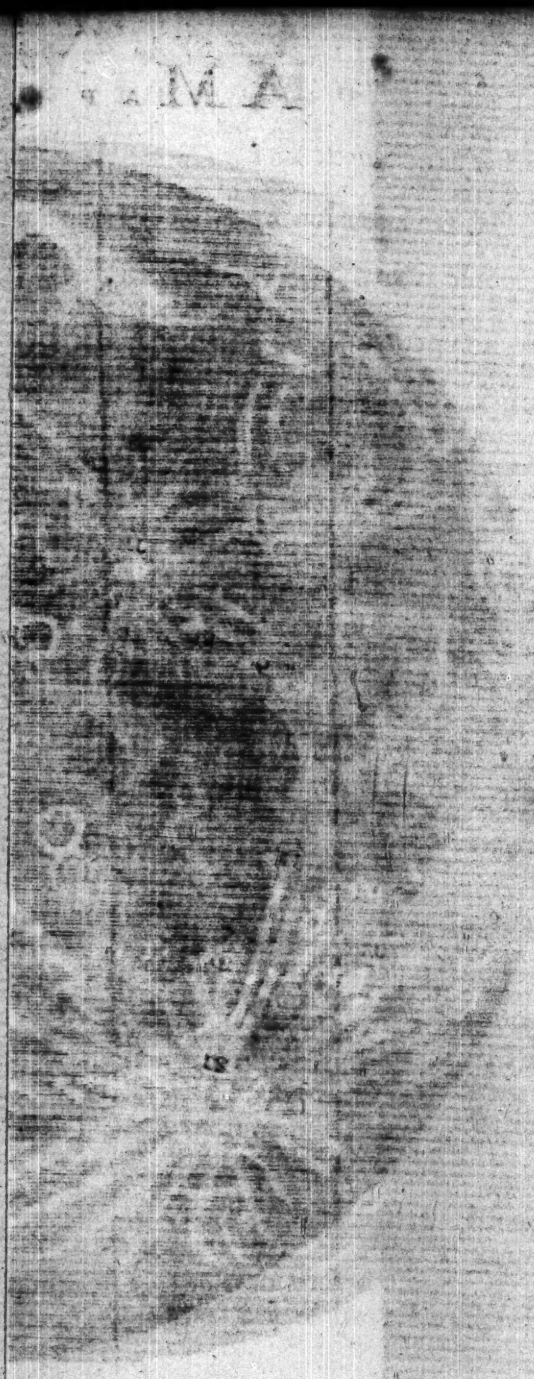


Published as the Act directs, by S. Dunn, No 6.

F THE MOON.



nn, N°. 6. Clement's Inn, February 14. 1774.



Of the Lunar Eclipſe, July 30. 1776

The mean Solar Time was then before Solar Time 5m. 54s. the Clock 23 Seconds after mean Solar Time. So the Clock had gained 9 Seconds, in 24 Hours, and 4 Seconds from the Forenoon to the Beginning of the Eclipſe; and therefore, 5m. 48s. ſubtracted from any of the foregoing Obſervations, gives the Solar Time for Greenwich.

70th. Here are 52 Obſervations made during the continuance of one Lunar Eclipſe, to any one of which a correſpondent one being made any where by Land or Sea, ſo that the Error in both is not more than 20 Seconds of Time, determines the Longitude to 5 Miles near the Equinoctial, and for 10 Seconds of Time 2½ Miles; an Error, which, (as hath been ſhewn) may be too ſmall to be diſcerned in the Practice by the Sun and Moon or Zodiacal Stars, and which is much leſs than that ariſing through an unfavourable State of the Air, in Obſervations of Jupiter's firſt Satellite with the beſt Teleſcopes. And that this ſame Eclipſe may be applicable to that Exactneſs, will appear by taking a Medium of the three Obſervations made of each of the principal Spots, and comparing it with the obſerved central Biſection, thus.

	Seconds of Time.		Seconds of Time.
Kepler	10 plus.	Kepler	2 plus.
Copernicus	10 minus.	Tycho	2 plus.
Plato	2 minus.	Copernicus	3 minus.
Tycho	1 plus.	Plato	10 minus.
Manilius	2 plus.	Manilius	2 plus.
Grimaldus	0.	Pliny	1 minus.

From theſe Compariſons, it appears, that the central Biſection of nine of theſe twelve Spots, was taken without erring two Seconds of Time, from a Medium of their Limbs or Extremes; and had the Error been double that Quantity, the Error in the Longitude deduced therefrom, would have been at the Equinoctial but one Geographical Mile.

71ſt. As the Eclipſe before mentioned was viſible to a great Part of the known World, probably it may have

have been well obſerved in this Manner at the principal Towns in England, Scotland and Ireland, and the Geography of thoſe Maps may have been thereby ſo perfected, as ever hereafter to need no Correction. The like may be ſaid for Places beyond the Seas. But it's Uſe in drawing Variation Charts of the Oceans is manifeſt; for at Sea, wherever the Time at the Ship was well aſcertained, and any of the Particulars obſerved and compared with the Obſervation, the Longitude of that Place at Sea was thereby well determined; and the Variation of the Needle taken at the ſame Place and Time, of the greateſt Uſe in delineating accurate Variation Charts of the Oceans.

XXIII. *Of Accurate Variation Charts.*

72d. But whenever the angular Diſtance of the Sun and Moon, or Moon and Zodiacal Stars, can be more accurately taken by Land and Sea; Variation Charts of the Oceans may be proved and drawn, if required, by the following Method.

At any Place whoſe Latitude and Longitude are well aſcertained, let the angular Diſtances of the Sun and Moon in the Day-time, and of the Moon's Limb and proper Zodiacal Stars, be taken at equal Intervals of either mean ſolar or ſiderial Time, as frequently as Obſervations can be made; one at the End of every three or ſix Hours, may be ſufficient. An Inſtrument for ſuch a Purpoſe, ſhould not be overlarge and cumberſome, but have a ſmall Teleſcope adapted to it, and be well mounted on a ſmall Pedeſtal; and then thoſe Diſtances of Stars may be taken thereby, without erring more than 15 Seconds of a Degree. I cannot approve of the Hadley's Sextant as an Inſtrument proper for ſuch a Uſe, on Account of many Reaſons that might be given; but if no better were made than that mentioned at Article 213 of my Practical Aſtronomy, it would anſwer within the Limits above mentioned.

73d. Obſervations being thus made by the Aſtronomer on Land, it would remain only to take the angular Diſtances of Sun and Moon, or Moon and Zodiacal Stars,

Of Accurate Variation Charts.

Stars, either at Sea or on any Coaſt, by the Method now practiſed, with Hadley's Sextant; and for the ſame Inſtant to determine the Time at the Ship and the Variation of the Needle, and to inſert theſe in the Ship's Journal for Compariſon with Obſervations at Home. By theſe, the Time at the Obſervatory and the Time at the Ship would be eaſily had, much in the ſame Manner as it is now found by the Lunar Method, but without any ſuch Errors as ariſe through Imperfection in the Lunar Tables; conſequently, the Longitudes of thoſe Places, either on the Coaſt or at Sea, would be had, alſo the Variation of the Magnetic Needle at thoſe Places; from which the Variation Charts may be drawn and corrected, as often as ſhall be thought neceſſary.

74th. Such a Method of obſerving need not be continued but a ſmall Time, and a ſufficient Number of Obſervations may be procured for drawing thoſe Charts, but a few ſuch will be enough for that Purpoſe, if the Theory be applied, by which I have drawn the New Variation Charts lately publiſhed. This Method of obſerving, is peculiarly adapted for determining the Longitudes of remote Places near the Sea Coaſt, alſo of Iſlands in the midſt of the Oceans, and will ſet aſide the Errors which are apt to be introduced through Imperfection in the Lunar Tables, and make the Concluſions equally certain, as though Predictions could be made to the greateſt Perfection. In this Method, the real angular Diſtances of the Sun and Moon, will always determine the Moon's Place, without an Error ariſing by a Compariſon of their equatorial Motions.

XXIV. *Of Longitude by the Ship's Reckoning.*

75th. Having ſhewn in a particular Manner how the Longitudes of Places may be determined on Land, in the Treatiſe of *Aſtronomy* before mentioned; alſo, having confirmed the Utility of correſpondent Obſervations on Lunar Eclipſes by that before mentioned; alſo having given an additional Method, whereby the Longitudes of Places near the Sea Coaſt and in the midſt

of the Oceans, may be found by correſpondent obſerved Diſtances of the Sun and Moon, or Moon and Stars; it is next to be conſidered, how the Longitude may be found at Sea, by each of the four principal Methods, what are the Calculations, and the Reſult of the Obſervations.

76th. In the Practice of the firſt Method, namely, that by the Ship's Reckoning, it is requiſite that the Navigator be perfectly acquainted with all the Parts of what is commonly called Practical Navigation, and in particular, that he be able to practiſe the nicer Parts thereof, ſuch as taking the Latitude and Variation, aſcertaining the Leeway and Currents, judging whence the principal Errors may have ariſen, and what kind or quantity or allowance ſhould be made for correcting them, how the Computations are to be made, and of the Reſult to be determined. The Principles on which theſe Concluſions depend, are copious and extenſive, they are no leſs than the Elements of Arithmetic, Geometry, Aſtronomy and Geography, and therefore may be conſidered as no inconſiderable Part of ancient and modern Learning.

77th. The various Parts of this Science have been fully conſidered in my Treatiſe of *Practical Navigation*; in which, it hath been endeavoured to point out the moſt eſſential Things to be obſerved for bringing this Subject to it's greateſt perfection, omitting nothing of Importance introduced by former Writers, and correcting Errors which have long Time been in the Practice and unobſerved. To that Treatiſe, I refer, for this firſt Method of finding Longitude at Sea, by the Ship's Reckoning or Courſe and Diſtance ſailed. It may not be improper to add here, that this Method will always ſucceed beſt, when the following Advantages attend the Practice, namely; the Courſe ſailed is as correct as poſſible; the Diſtance meaſured is nearly true; and the Latitude is taken frequently and applied in the Correction of the Longitude.

78th. The Error ariſing from ſailing on a Courſe different

different from what is ſuppoſed, may often be diſcovered, when the Ship hath kept for a conſiderable Time on one and the ſame Courſe, and by help of the obſerved Latitude, finds her Place to have been erroneouſly inferred from the Reckoning. But for meaſuring the Diſtance ſailed, if the Log line be thought imperfect, the following Method may be uſed, which I tried more than 20 Years ſince on the River Thames, and found it to anſwer Expectation.

Seeing that ſome Bodies will ſwim in Water and others will ſink to the Bottom; it follows from the Principles of Hydroſtatics, that a ſmall Ball of any Subſtance ſpecifically heavier than Sea Water, will ſink below its Surface; and if its ſpecific Gravity be greater than that of Water, but properly proportioned to it, the Ball, when immerſed in the Water, may be ſuſpended by a Thread perpendicularly, at Reſt, and with a Power leſs than that of it's own Weight in Air; but if the Hand holding the Thread is moved, the Ball will recede, and the Thread will make an Angle with the horizontal Perpendicular.

79th. Being provided with one of theſe ſmall Balls, whether of heavy Wood, or of Braſs, Lead, or a ſmall globular Shell filled with Quickſilver; the Method of uſing it in determining the Motion of the Ship at Sea, is thus.

Having fixed on two Points of Land, whoſe Diſtance is well aſcertained in Nautical Miles, and faſtened the Thread or Cord to the Centre of a ſmall Arch divided into Degrees, with its Radius Perpendicular to the Horizon, let the Angle of Receſſion, from a Perpendicular Direction, be taken whilſt the Ship is ſailing from one Place to the other, and the Time elapſed be meaſured by a Watch; ſo will the Velocity anſwering to that Receſſion, be determined. And the like for other Degrees of Receſſion, from which, a ſmall Table may be made, ſhewing the intermediate Degrees and their reſpective Motions, to any Preciſion, and ſuch a

Table, with the Inſtrument, are to be uſed for ſhewing the Ship's Motion per Hour, at Sea and during a Voyage.

Otherwiſe, the Experiment may be made by running a good Length at Sea near one and the ſame Meridian, with nearly the ſame Velocity, and the Diſtance found by the obſerved Latitudes of both Places, for this Diſtance with the Time of Sailing it, will give the Number of nautical Miles per Hour; and the like for any other Velocity.

XXV. *Of Longitude by Help of the Pocket Watch.*

80th. When the Longitude at Sea is to be found by the Uſe of a Pocket Watch, it ſhould be carefully tried for a ſufficient Time before the Voyage begins, to diſcover whether it may be expected to keep equal Time at Sea or not; and alſo, what it may gain or loſe on Equal Time in 24 Hours whilſt it is thus on Trial, becauſe that Allowance is to be made whilſt it is going at Sea. The beſt Method of trying it on Land, is either by obſerved Tranſits of the Sun or Fixed Stars, concerning which I have given full Inſtructions in my *Practical Aſtronomy*, and illuſtrated the ſame with Examples.

In the Uſe of ſuch a Watch, the greateſt Care ſhould be taken of it whilſt it is in Uſe, to prevent any Accidents from hurting it. Before the Beginning of the Voyage it ſhould be clean and brought to a regular Rate of going, and be as much as poſſible kept in the Fob or Side pocket, to give it always the ſame Degree of Warmth, and prevent any ill Effect through the Change of Climate or Air.

If the Voyage is to be a long one, and the Ship a long Time from Land whoſe Longitude is known, there ſhould be the greater Care in adjuſting the Watch, to make it keep Equal Time, and know it's Error; becauſe, in the Uſe of this Machine, the Error in the Longitude found, ariſeth from the Length of Time it hath been ſuppoſedly keeping Equal Time. For Inſtance, if a Watch err from its Equal Time one Minute in a Week, that may produce an Error of 15 Miles in Longitude

Longitude; but if it err four Minutes in a Month, that may produce an Error of 60 Miles in Longitude, although the Watch hath gone on at the ſame Rate.

81ſt. If the Watch cannot be ſet by an Obſervation of the Sun's Meridian Tranſit, it may be, by either a ſingle or a double Altitude of the Sun, the Manner of doing which is ſhewn in the forementioned Treatiſes of *Navigation* and *Aſtronomy*, in the former of which, under that Head, are all the Inſtructions neceſſary for determining the Latitudes of Places at Sea, the Amplitude, Azimuth, and Time at the Ship, under the neceſſ Corrections relative to thoſe Subjects; and which, belonging chiefly to Practical Navigation, are ſuppoſedly underſtood by the Navigator, as a Qualification preparatory to the Study and Practice of the Longitude at Sea, by any of the four Methods before mentioned. Nevertheleſs, as this Method depends ſo much on the Examination of a Time Keeper, it is proper here to ſhew the Method of making this Calculation by an Example.

Suppoſe March 26, 1780, before Noon, at Sea in Latitude 50d. 0m. N. and by Eſtimation 30d. Longitude weſt from London; at 8h. 36m. per the Ship's Clock, or by a Pocket Watch, the Altitude of the Sun's lower Limb is obſerved 26d. 24m. and the Height of the Eye 14 Feet above the Surface of the Sea. Required, how much the Watch is either before or after Solar Time.

For the Solution of this Problem, if the Sun be near the Equator, the eſtimated Longitude of the Ship and nearly the Time of the Day there, muſt be known in order to take out the Sun's true Declination, and thereby get the true Polar Diſtance; but when the Sun is near either of the Tropics, there is no need of this, as the Declination continues nearly the ſame during 24 Hours.

In the preſent Inſtance, by comparing the eſtimated Longitude and Time at the Ship, the eſtimated Time at London is 10h. nearly. The Declination for Noon at London

at London is 2d. 34m. North; and allowing for the daily Change of Declination, it is 2d. 32m. for the Time of Obſervation at the Ship, and the Polar Diſtance 87d. 28m. for *s p* one of the three Sides of an oblique ſpherical Triangle. The Sun's Semidiameter is 16m. additive; the Dip .4m. and Refraction 2m. both ſubductive; ſo the true Altitude of the Sun's Centre is 26d. 34m. and Co-altitude 63d. 26m. for *s z* another Side of the ſame Triangle. The Co-latitude is 40d. om. for *p z* the third Side of the ſame ſpherical Triangle. And the Operation will be thus

	d.	m.
Co-latitude, always under 90. -	40	—
Polar Diſtance, under or above 90d. -	87	28
Co-altitude, always under 90d. -	63	26
Sum of theſe three Sides - -	190	54
Half Sum of theſe three Sides -	95	27
Co-alt. ſub. from half Sum. *Remainder* -	32	1

	d.	m.		Log.
Co. ar. of Colatitude	40	—	Log.	0.1919
Co. ar. of Polar Diſtance	87	28	——	0.0004
Sine of half Sum -	95	27	——	9.9980
Sine of Remainder -	32	1		9.7244
Sum of theſe four Logarithms -				19.9147
Half Sum of them				9.9573
Coſine Arch of the Half Sum			24d.	58m.
Double Coſine Arch			49d.	56m.
Double Coſine Arch, *in Time*			3h. 19m.	44s.
Solar Time of the Morning -			8h. 40m.	16s.
Mean Solar Time before Solar -			oh. 5m.	34s.
Mean Solar Time of Obſervation -			8h. 45m.	50s.
Time by the Watch -			8h. 36m.	—s.
Watch before Solar Time -			oh. 4m.	16s.
Watch after Mean Solar Time -			oh. 9m.	50s.

In this Operation, when the Co. ar. (or Complement arithmetical) is taken out, it is the Log-coſecant, with the Index diminiſhed by 10. Alſo, when the Co.ar. or Sine of any Arch more than 90d. is to be taken, the Supplemental Arch to 180d. is uſed in it's ſtead.

82d. In the Uſe of this general Problem, it will be wanted

wanted to have the Sun's Altitude as correctly as poſſible, which when the Ship happens to be near Land will not always be practicable; and at Sea, the Horizon will often be too ill defined for taking the Altitude to the Accuracy that is wanted. But a Remedy to the former of theſe two Caſes will ariſe from a judicious Application of Hadley's Sextant, when the Horizon is viſible on that Point which is oppoſite to the Bearing of the Sun; for the Obſerver may turn his Back to the Sun, and bring the Sun's upper Limb to ſweep that Horizon, but the Semidiameter will, in this Caſe be additive, to get the Co-altitude of the Centre. This can be practiſed by the Sextant when the Co-altitude is not more than 30 Degrees; but if the back Obſervation be made with Hadleys's Octant, this Defect may be ſupplied down to the Horizon, however I think the Truth would be much more eaſily found, by the New Inſtrument mentioned, page 73, of my *Practical Aſtronomy*, and that there is wanted that Inſtrument, for aſcertaining many things of Importance in Navigation.

83d. The Time Keeper having kept mean Solar Time during the Voyage, and the Solar Time at the Ship being found by Obſervation, their Difference is the Longitude. Thus, if the Solar Time at the Ship be 8h. 36m. when the Equation is as beforementioned, and the mean Solar Time by the Watch 8h. 45m. 50s, the Solar Time by the Watch will be 8h. 40m. 16s. from which take 8h. 36m. gives oh. 4m. 16s. or 1d. 4m. for the Ship's Longitude weſt from the Place where the Watch was ſet to mean Solar Time.

84th. This is the Method (which hath been ſo much wiſhed for and deſired) of finding the Longitude at Sea by Help of a Time Keeper; which, if the Machine could be depended on, and no other Corrections were required, would be of the greateſt Uſe in Navigation. But it frequently happens, that ſuch Machines will err from an equal Rate of going after they have been adjuſted thereto with the greateſt Diligence and Care, that ſuch ſmall Errors ariſe from Acoidents and Circumſtances

ſtances which are not eaſily to be foreſeen, and that their Effect is not always diſcoverable, but unobſerved by the Navigator whilſt he is purſuing his Voyage. It is therefore requiſite to watch the Motions of ſuch Machines and diſcover the Errors as ſoon as poſſible after they ariſe. If the Machine requires to be turned up once in every 24 Hours, it ſhould be carefully tried whether it will keep Equal Time during that Interval; to which, if there be added a ſameneſs of Poſition, an equality of Heat, and Care to preſerve it from ſudden ſhocks and other Accidents, the Merit of the Machine, will appear at the end of every ſuch 24 Hours trial. The like, if it be made to turn up at the End of 2 Days, or any other Time ſuited for Uſe at Sea; and the Subdiviſions of the intermediate Hours, may be ſhewn by a good Spring Clock or Pocket Watch, either of which can now be made ſufficiently correct for that Purpoſe, although they may be but rarely found ſufficient for keeping correct Time during a long Voyage.

85th. From Conſiderations like theſe, it was, that more than 25 Years ago I applied to a certain reſpectable Board, requeſting that ſome ingenious Watch-maker might be ordered to make ſuch a Machine under my Inſpection, as the Obſervations which I had made on the going of Machines, confirmed the Poſſibility of making one that would meaſure large Intervals of Equal Time and that would not be much diſturbed by the Motion of a Ship at Sea. This Propoſal was heard, and I was ſent to a well known and ingenious Aſtronomer, to whom I deſcribed the Conſtruction of ſuch a Machine; concluding therefrom, that it could not poſſibly have any ſenſible Difference of either Friction or any other Thing affecting it, more during one whole Revolution of the Machine, than during another, that the quickneſs of the Vibration of the Pendulum Spring, was that chiefly by which the Error ariſing from the Motion of the Ship could be overcome, and that a Method of allowing for the Effect of Heat and Cold on the Machine, being deduced from Experiments, would

finiſh

A TABLE OF EQUAT

Shewing the SUN's Southing,

Month Day	JANUARY 1776 1780	77 81	78 82	79 83	FEBRUARY 1776 1780	77 81	78 82	79 83	177 178
	h ″ ″	″	″	″	h ″ ″	″	″	″	h
1	12. 4. 1	21	14	7	12.14. 3	8	6	4	12.
2	12. 4.28	49	42	35	12.14.10	15	14	12	12.
4	12. 5.24	44	38	31	12.14.23	26	25	24	12.
6	12. 6.17	37	31	24	12.14.32	34	33	32	12.
8	12. 7. 9	28	22	16	12.14.37	39	38	38	12.
10	12. 7.59	17	11	5	12.14.40	40	40	40	12.
12	12. 8.46	4	58	52	12.14.40	39	39	39	12.
14	12. 9.31	28	42	37	12.14.36	34	35	35	12.
16	12.10.14	29	24	19	12.14.30	27	28	29	12.
18	12.10.53	7	3	58	12.14.21	16	17	19	12.
20	12.11.30	43	38	34	12.14. 9	3	5	6	12.
22	12.12. 3	15	11	7	12.13.54	47	49	51	12.
24	12.12.34	44	41	37	12.13.36	29	31	34	12.
26	12.13. 1	10	7	4	12.13.17	9	11	14	12.
28	12.13.25	33	30	27	12.12.55	46	49	52	12.
30	12.13.45	52	50	48	----------	----------	----------	----	12.
	MAY				JUNE				
1	11.56.47	49	51	53	11.57.27	25	23	21	12.
2	11.56.40	42	43	45	11.57.36	34	32	30	12.
4	11.56.27	29	30	32	11.57.50	54	52	49	12.
6	11.56.17	18	20	21	11.58.17	15	12	10	12.
8	11.56. 9	10	11	12	11.58.40	37	34	32	12.
10	11.56. 4	4	5	6	11.59. 3	1	58	55	12.
12	11.56. 0	1	1	2	11.59.27	25	22	19	12.
14	11.55.59	1	0	0	11.59.52	49	46	44	12.
16	11.56. 1	1	1	0	12. 0.18	15	12	9	12.
18	11.56. 4	4	3	3	12. 0.44	41	38	35	12.
20	11.56.10	9	9	8	12. 1. 9	6	3	0	12.
22	11.56.18	17	16	15	12. 1.35	32	29	26	12.
24	11.56.28	27	25	24	12. 2. 1	58	55	52	12.
26	11.56.40	39	37	36	12. 2.26	23	20	17	12.
28	11.56.54	52	51	49	12. 2.51	48	45	42	12.
30	11.57. 9	8	6	4	12. 3.14	12	9	6	12.
	SEPTEMBER				OCTOBER				
1	11.59.32	36	41	46	11.49.24	29	33	38	11.
2	11.59.13	18	22	27	11.49. 6	10	15	19	11.
4	11.58.34	39	44	49	11.48.30	34	38	43	11.
6	11.57.55	59	4	9	11.47.55	59	3	8	11.
8	11.57.14	19	24	29	11.47.22	26	30	34	11.
10	11.56.33	38	43	48	11.46.51	55	58	2	11.
12	11.55.52	57	2	7	11.46.22	23	29	22	11.
14	11.55.10	15	20	25	11.45.55	58	1	4	11.
16	11.54.29	33	38	43	11.45.30	33	36	39	11.
18	11.53.46	51	56	1	11.45. 7	10	12	15	11.
20	11.53. 4	9	14	19	11.44.47	49	52	54	11.
22	11.52.23	28	33	38	11.44.30	32	34	36	11.
24	11.51.42	47	51	56	11.44.15	17	18	20	11.
26	11.51. 1	6	11	16	11.44. 3	5	6	7	11.
28	11.50.22	26	31	36	11.43.55	56	56	57	11.
30	11.49.43	48	52	57	11.43.49	49	50	50	11.
	1784	85	86	87	1784	85	86	87	1
	1788	89	90	91	1788	89	90	91	1
	1792	93	94	95	1792	93	94	95	1
	1796	97	98	99	1796	97	98	99	1

☞ The Hours & Minutes read with the Second

Published according to Act of Parliament January 24. 1774 By Sam

UATION OF TIME;

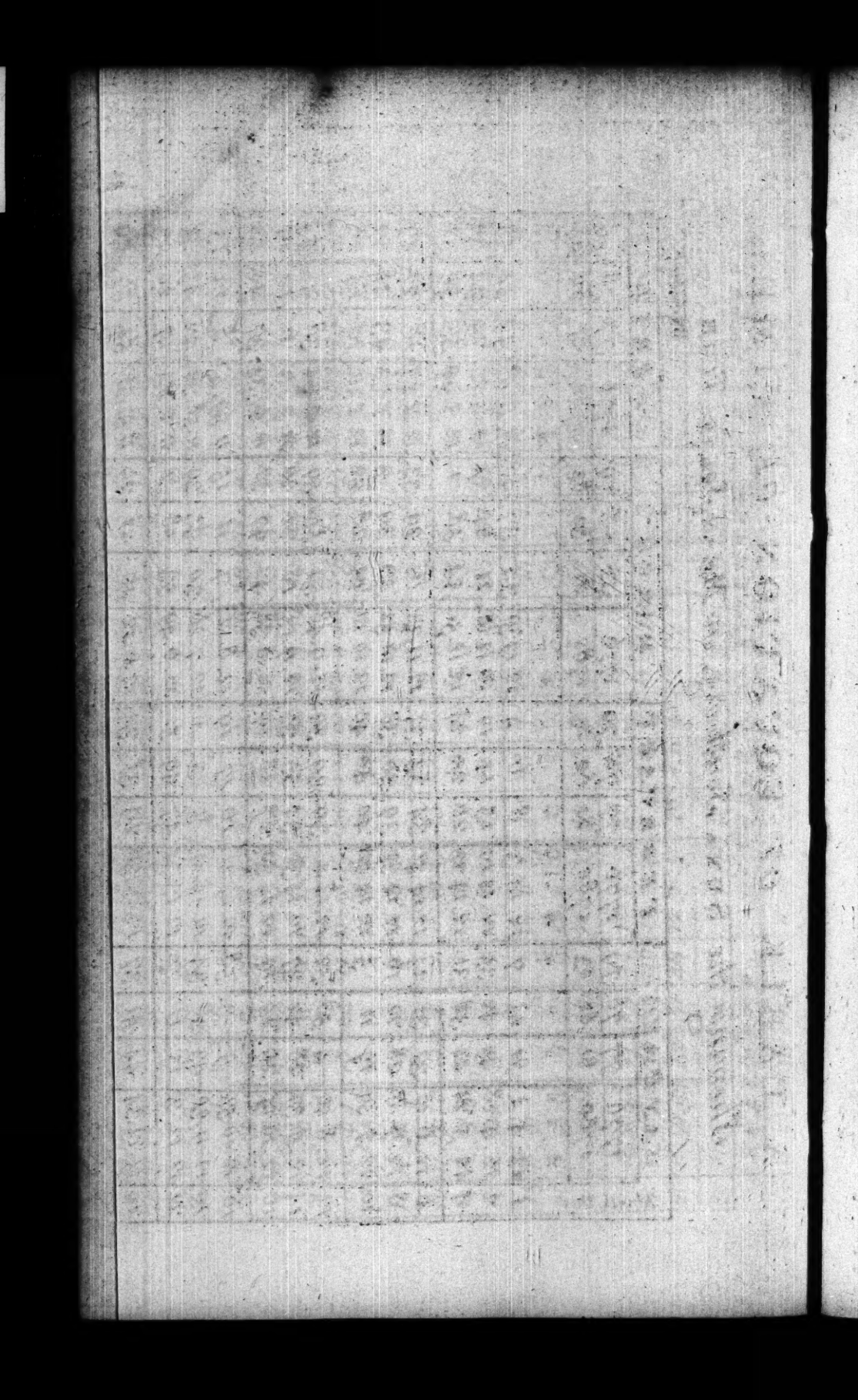
hing, in MEAN SOLAR TIME.

By S. Dunn.

Y.	MARCH				APRIL			
79	1776	77	78	79	1776	77	78	79
83	1780	81	82	83	1780	81	82	83
"	h " "	"	"	"	h " "	"	"	"
4	12.12.31	34	37	40	12. 3.45	47	52	57
12	12.12.18	21	24	27	12. 3.25	29	34	38
24	12.11.51	54	58	1	12. 2.49	53	58	2
32	12.11.23	26	29	33	12. 2.14	18	22	27
38	12.10.52	56	59	3	12. 1.39	43	48	52
40	12.10.20	24	28	32	12. 1. 6	10	14	18
39	12. 9.48	52	55	59	12. 0.34	38	41	45
35	12. 9.13	18	22	26	12. 0. 3	6	10	14
29	12. 8.38	42	47	51	11.59.33	37	40	44
19	12. 8. 2	7	11	15	11.59. 5	8	12	15
6	12. 7.26	30	35	39	11.58.39	42	45	48
51	12. 6.49	53	58	2	11.58.14	17	20	23
34	12. 6.12	16	21	25	11.57.51	54	57	59
14	12. 5.34	39	43	48	11.57.30	33	35	38
52	12. 4.57	2	6	11	11.57. 11	14	16	18
...	12. 4.20	24	29	33	11.56.55	56	58	0
	JULY				AUGUST			
21	12. 3.26	23	21	18	12. 5.50	51	52	53
30	12. 3.37	35	32	29	12. 5.46	47	48	50
49	12. 3.59	57	54	52	12. 5 36	37	39	40
10	12. 4.19	17	15	13	12. 5.24	25	27	29
32	12. 4.38	36	34	32	12. 5. 9	11	13	15
55	12. 4.56	54	52	50	12. 4.52	54	56	58
19	12. 5.11	9	8	6	12. 4.32	35	37	40
44	12. 5.25	23	22	20	12. 4.10	13	16	19
9	12. 5.37	35	34	33	12. 3.47	50	53	56
35	12. 5.46	45	44	43	12. 3.21	24	28	31
0	12. 5.54	53	52	52	12. 2.53	57	0	4
26	12. 5.59	59	58	58	12. 2.24	27	31	35
52	12. 6. 2	2	2	2	12. 1.52	56	0	4
17	12. 6. 3	3	3	3	12. 1.19	24	28	32
42	12. 6. 1	1	2	2	12. 0.45	49	54	58
6	12. 5.57	58	58	59	12. 0. 9	14	18	22
	NOVEMBER				DECEMBER			
38	11.43.46	46	47	47	11.49.43	37	31	25
19	11.43.46	46	46	46	11.50. 6	0	55	49
43	11.43.49	48	48	47	11.50.55	49	43	37
8	11.43.54	53	52	51	11.51.47	40	34	27
34	11.44. 3	2	1	59	11.52.40	33	27	20
2	11.44.16	14	12	11	11.53.35	28	21	14
22	11.44.31	29	27	25	11.54.31	24	17	10
4	11.44.50	48	45	43	11.55.29	22	15	7
39	11.45.13	10	7	4	11.56.28	20	13	6
15	11.45.39	35	32	29	11.57.27	20	12	5
54	11.46. 8	4	1	57	11.58.27	20	12	5
36	11.46.40	36	32	28	11.59.27	20	12	5
20	11.47.16	11	7	2	12. 0.27	20	12	5
7	11.47.54	49	44	40	12. 1.27	20	12	5
57	11.48.36	30	25	20	12. 2.26	19	12	4
50	11.49.20	14	9	3	12. 3.24	17	10	3
87	1784	85	86	87	1784	85	86	87
91	1788	89	90	91	1788	89	90	91
95	1792	93	94	95	1792	93	94	95
99	1796	97	98	99	1796	97	98	99

Seconds which do not belong to Bissextile Year.

By Samuel Dunn Maiden Lane, Covent Garden, LONDON.



finiſh it for Uſe at Sea. The Anſwer to me was, that another Perſon had made a Machine which had been tried at Sea, and wanted only to be ſuſpended by friction Rollers in a Frame, to take off the Motion of the Ship; that it went by a ſlow Motion contrary to what I had imagined, was large, with it's Pendulum moving ſlowly in a vertical Poſition, and that it would be improper to ſet another Perſon at Work about that Subject. About 9 Years after this, I heard that the Principle of a ſlow Motion was laid aſide and by introducing a quick one in it's ſtead, a Machine of a ſmall Size was made very little different in many Reſpects from a common Watch, yet ſufficient for the Longitude. Since that time, I have not heard of any Time-keepers intended for the Longitude, but have had this Property. Mr. Harriſon, ſpeaking of his Time-keeper, page 14 ſays, "The grand Principle of the Watch is "that of giving the greateſt Motion poſſible to the "Balance with a given Force." And I have been informed by an eminent Watch-maker, that the beſt Pocket Watches which have been made ſince Watch-work received it's capital Improvements, do beat near 18 thouſand Beats an Hour, or near 5 Beats in a Second of Time, which confirms the Principle before mentioned.

XXVI. *Of Longitude by the Sun, Moon and Zodiacal Stars.*

86th. Having ſhewn what are the principal Things to be attended to in the two firſt Methods of finding the Longitude at Sea; the third Method comes to be conſidered, and in a more extenſive Manner than either of the former in this Work, as it exerciſeth a greater Variety of Science relative to the Principles which it is founded, the Obſervation and Calculation. The principal Sections of the general Introduction prefixed to this Work, which relate to the ſecond Method, are thoſe concerning the Equation of Time, the general Properties of Clocks and Watches, and the Errors ariſing in them and the Uſe of other Inſtruments. But in this third Method (in order to a good Underſtand-

ing of the Subject) it is requiſite to be acquainted with that general Introduction throughout, except the latter Part thereof concerning the Variation of the Magnetic Needle, which alſo may be of ſome particular Uſe, when this third Method is applied in finding the Longitude at Sea.

87th. The Inſtruments and Books uſed in this Method are, *Hadley's Sextant* or *Octant*, the *Ephemeris* for the current Year in which the Obſervation is to be made, *Logarithms*, *Logarithmic Sines*, *Tangents* and *Secants*; to which the Obſerver muſt be able to add a Competency of Skill and Ability to make the Obſervations and Calculations.

88th. Being provided with theſe Neceſſaries, at Sea, two things muſt be found to determine the Longitude of the Ship; firſt, what Hour, Minute and Second of the Day or Night in Solar Time, it is at the Place for which the Ephemeris is made, when he makes the Obſervation at the Ship; and ſecondly, what Hour Minute and Second of Solar Time of the Day or Night, it is at that ſame Inſtant, at the Ship; and the Difference of theſe two turned into Degrees, by allowing 15 Degrees to an Hour of Time, gives the Longitude of the Ship from the Place for which the Ephemeris was made. If the Time at the Ship is greateſt, the Ship is in Eaſt Longitude, otherwiſe Weſt.

89th. If the Obſervation be made in the Day time, the Altitude of the Sun's Centre, that of the Moon's Centre and the Diſtance of the Sun and Moon's neareſt Limbs, all for the ſame Inſtant, are the only things to be obſerved; but if the Obſervation be to made by Night, ſuch one of the Zodiacal Stars as is to be uſed for that Day, it's Altitude is to be taken, alſo the Altitude of the Moon, and the Diſtance from the Star to the enlightened Limb of the Moon for the ſame Inſtant. Theſe are called the three cotemporary Obſervations.

90th. In order to make theſe Obſervations to Advantage, it ſhould be conſidered what Errors may ariſe under unfavourable Circumſtances, and therefore as much

much as poſſible to avoid any ſuch. For Inſtance, if the Sun happens to be very near the Meridian of the Place of Obſervation, it will frequently be very difficult to infer the correct Time at the Ship, on Account of the ſlow change in the Altitude of the Sun. If the Sun or Moon happens to be near the Horizon, they may be ſo affected with unexpected Refraction in different Climates and at different Seaſons of the Year, that the obſerved Diſtance of the Limbs may not be ſo correctly taken, as appears by the Inſtrument. For the Night Obſervation, it will be moſt convenient to have a *Watch*, which having been ſet to Solar Time in the Day, ſuppoſedly carries Time 'till Night, but if that ſhould err in it's Rate of going, or ſhould not have been ſet right, or a proper Allowance be not made for the Removal of the Ship in the Interval, an Error may thereby be produced, which together with other Errors, may render the Concluſion the more imperfect. Or in general, it may be ſaid, the Obſervations for the Longitude by this Method, are not eaſily made with that Accuracy that is deſirable, but by Perſons who can exert their Care and Attention in a particular Manner, and are acquainted with the Principles on which it is founded; ſome of which are in the general Introduction, and the reſt are as follows.

Problem I. To find on what part of the ſmall Mirror of the Sextant, the Objects muſt be ſeen when they are united.

Rule. Meaſure the ſame Diſtance from the Plane of the Inſtrument to the unquickſilvered Part of the ſmall Mirror, as the Eye-hole is from the Plane of the Inſtrument; and it gives the Place required. If the Eye-hole be moveable, it is beſt to ſlide it more or leſs diſtant from the Plane of the Inſtrument, until it is the ſame Diſtance as the Line between the quickſilvered and unquickſilvered Part, and then the Objects are to be united on that Line.

Problem II. To adjuſt the Sextant horizontally.

Rule. Hold it horizontally, and by the Screw on

the Foot of the ſmall Mirror, move it forward or backward, until the Horizon of the Sea, or any ſtraight Line at a Diſtance, appears one continued ſtraight Line, both at right and left of the Mirror, and on the quickſilvered Part thereof.

Problem III: To adjuſt the Sextant vertically.

Rule. Set the Beginning of the Diviſions on the Index exactly at the Beginning of the Degrees on the Arch of the Sextant; and then, holding the Inſtrument as upright as poſſible, turn the ſmall Mirror about by the Lever on the Back of the Inſtrument, until the Horizon of the Sea, or any diſtant Object, appears united and making one perfect Object, partly on the quickſilvered and part on the unquickſilvered Glaſs or ſmall Mirror. This Adjuſtment may be alſo made by the Sun in the Day-time, or in the Night by a bright Star.

Problem IIII. To unite or join the Objects on the ſmall Mirror of the Sextant.

Rule. Always the brighteſt Object is brought to the fainteſt; therefore, the Sun is brought to the Moon, and the Moon is brought to a Star. If the Light be too ſtrong, the dark Glaſs muſt be turned down to take off the ſuperfluous Light. There are four principal Poſitions which the Inſtrument may require in this Obſervation. 1ſt. To bring the Object down. 2d. To carry it upward. 3d. To bring it to the right. 4th. To carry it to the left. In each of which the Inſtrument muſt be turned ſo as to produce that Effect. The like is to be obſerved for other Poſitions. But in all of them, the moveable Object muſt be made to ſweep the immoveable one, ſo that the Contact or Touch be as correct as poſſible, and at the ſame time the Junction muſt be on the Part of the ſmall Mirror before deſcribed.

Problem V. To find the Latitude of a Place.

This Problem, belonging to *Practical Navigation*, is fully diſcuſſed in the Treatiſe on that Subject. The moſt uſual Practice is by the Sun's Meridian Altitude; but the ſame thing may be effected by the Meridian Altitudes of the Fixed Stars of the firſt Magnitude, when the

the Horizon is favourable, and the Stars Declinations are known.

Problem VI. Having the Latitude of the Place, the Declination of the Sun, and the Altitude of the Sun's lower Limb, above the Horizon of the Sea; to find the Solar Time, before or after Noon.

This Problem hath been already exemplified in the Method of finding the Longitude by help of a Time-keeper, and in the forementioned Treatiſe, it is alſo ſhewn how to find this and ſeveral other Things, by two Altitudes and the obſerved elapſed Time.

Problem VII. To find how much a Watch gains or loſes on mean Solar Time, whilſt the Ship is purſuing her Voyage at Sea.

This Problem is contained in that before exemplified in finding the Time at the Ship. Which being done at the End of one, two or more Days, and the Difference of Longitude in that Interval being allowed, will give the Gain or Loſs required. But if any Error ariſeth for want of knowing the correct difference of Longitude by the Reckoning, it will be imperfectly allowed for and the Error be undiſcovered.

Problem VIII. To ſet a Watch to Solar Time, by having the Declination and Right Aſcenſion of a Star, the Latitude of the Place, and the Altitude of the Star above the Horizon of the Sea.

In this Problem, the ſame Proceſs is to be made as that for the Sun, until the equatorial Diſtance of the Star from the Meridian of the Place of Obſervation is found; then, the equatorial Diſtance of the Star from the Meridian, being compared with the equatorial Diſtance of the Sun from the Star, by help of the Star's Right Aſcenſion and the Sun's Right Aſcenſion for that Inſtant, gives the equatorial Diſtance of the Sun from the Meridian, and conſequently the Time of the Night required.

This Problem cannot be frequently practiſed at Sea for want of a clear Horizon; and otherwiſe, ſomewhat near the Longitude of the Ship muſt be known, to

determine

determine the correct Right Aſcenſion of the Sun at that Inſtant; but to make it general, there is wanted a correct Artificial Horizon to ſupply the imperfect Horizon of the Sea.

Problem IX. To know in what Direction the Ecliptic is, by having a Sight of the Sun and Moon only, in the Day-time.

Rule. Join the two Corners or Cups of the Moon by an imaginary ſtraight Line, which biſect at right angles and produce it to right and left through the Heavens, one toward the Sun, the other from the Sun, and it will be nearly the Poſition of the Ecliptic at that Time.

Problem X. To find the ſame as in the former Problem, by only a Sight of the Moon in the Night-time.

Rule. Biſect the Line joining the Cuſps and continue it as before, it will be the Poſition required, and in that continued Line the Zodiacal Stars will be found; but if they are without the Zodiac, the Eye muſt be directed a little to the North or South, according to the Star's Latitude.

Problem XI. To find a Parallel of Declination, on the Chart of Zodiacal Stars, equal to the Latitude of the Place.

Rule. Guide your Eye North or South of the Equator on the Chart, as many Degrees as is the Latitude of the Place, and it gives the Parallel required.

Problem XII. Having the Latitude of the Place and the Hour of the Day or Night; to find the Point of the Zenith, and what Stars are near the Zenith at that Time.

Rule. Find the Parallel of Declination as in the former Problem, and then guide your Eye along that Parallel, either to the Right or Left, as many Hours as it is either before or after Noon, and it gives the Point required; and the Stars near that Point, are then near the Zenith, but others to the Eaſt, Weſt, North or South thereof, ſomewhat in the Poſition as ſhewn by the Chart.

Problem

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Problem XIII. Having the Latitude of the Place, and the Hour of the Night, to form an Eſtimate, what may be nearly the Poſition of the Heavens at that Time.

Rule. Find the Place of the Zenith as in the two former Problems; and it will be ſhewn, not only what Stars are near the Zenith and Meridian, but alſo what Part of the Ecliptic is on the Meridian at that Time, and what Diſtance the Moon is therefrom, or any of the Zodiacal Stars.

Problem XIV. Having the Day of the Month, to find the Sun's Place, on the Chart of Zodiacal Stars.

Rule. Guide your Eye along the Ecliptic, until you come to ſuch a Day of the Month as is pointed out by the Months and their Days adjoining to the Ecliptic, and it gives the Place required.

Problem. XV To find what is nearly the Right Aſcenſion and Declination of the Sun, on a given Day.

Rule. Find the Sun's Place as in the former Problem, ſo the Right Aſcenſion is on the correſpondent Part of the Equator, and the Declination is the Diſtance from the Equator. But theſe are moſt correctly known by the Ephemeris or annual Tables.

Problem XVI. To find the Zodiacal Stars in the Night when the Moon doth not appear.

Rule. Find the Place of the Zenith and the Poſition of the Ecliptic, as in the former Problems; which being referred to the Heavens, the Zodiacal Stars will appear there, as they are in Miniature on the Chart. This is a very eaſy Method of finding the Stars that are to be uſed and diſtinguiſhing them from others, in the Practice of the Longitude at Sea in the Night, particularly thoſe which are too ſmall to be ſeen, when they happen to be near the full Moon.

Problem XVII. To diſtinguiſh the Planets from the Zodiacal Stars.

Rule. The Planets Mercury, Venus, Mars, Jupiter and Saturn, never appear far from ſome Part or other of the Ecliptic, and therefore near that Line they

are to

are to be ſought amongſt the Fixed Stars. It is a property of the Fixed Stars that they appear twinkling, except when they are much elevated or near the Zenith; but the Planets do not appear twinkling except they are near the Horizon. Mercury appears ſmall of a pale Colour and is never far diſtant from the Sun. Venus appears frequently large and ſplendid, but is never oppoſite to the Sun. Mars appears ſometimes ſmall at other times large as a Fixed Star of the ſecond Magnitude, and apparently makes Excurſion through all Parts of the Zodiac. Jupiter doth ſo likewiſe and generally appears large and refulgent. Saturn makes the like Excurſion, but generally appears pale, like a Fixed Star of the Second Magnitude. The periodic Times of theſe Planets compared with the periodic Time of the Earth round the Sun, and the Orbits of the two firſt being between the Sun and Earth, but of the three latter without the Earth's Orbit, their apparent Places are thereby more difficult to be predicted; and therefore whenever their Poſitions are required in the Zodiac, it is beſt to take out their Longitudes and Latitudes from an Ephemeris calculated for the current Year, and then they may be found in thoſe Places, according to the Method uſed for the Sun, Moon or Zodiacal Stars.

Problem XVIII. To find in what Part of the Zodiac the Moon is, the Month, Day and Hour being given; and thereby ſhew her Diſtance from and Poſition to the Sun and Zodiacal Stars.

Rule. By help of an Ephemeris ſhewing the Moon's Longitude every day at Noon, take out her Place for the Day of Obſervation, making allowance alſo for her Receſſion or Change of Longitude from the Noon to the given Hour; this gives the Moon's Place to be found on the Chart of Zodiacal Stars; at the right and left of which Place, in the Direction of the Ecliptic, the Sun and Zodiacal Stars will be, at their reſpective miniature Diſtances, as they are to be looked for in the Heavens.

This

Of Longitude by the Sun, Moon and Zodiacal Stars.

This Problem is of very great Uſe in finding the Longitude at Sea by the Moon; becauſe, it often happens, that in different Latitudes, the Ecliptic takes ſuch different Poſitions, at different times of the Night, real Proficients in Aſtronomy may ſometimes be at a loſs to find the Stars to be uſed, but by this Method, it is eaſy for all Times and Places.

Problem XIX. To find the compounded Effect of the Dip of Horizon, the horizontal Refraction and the ſpheroidal Figure of the Earth; in any Latitude, and at any Time of the Year, when the Sun is viſible.

Rule. Whilſt one obſerver is taking the Sun's Altitude above the Horizon facing the Sun, let another take the Diſtance of the ſame Edge of the Sun from the oppoſite Horizon, by bringing the Sun through the Zenith. The ſum of theſe two Arcs will make 180 Degrees, if there be no Effect from any other Cauſe; if they make leſs than 180 Degrees, take their Sum from 180 Degrees, and half the Remainder is the Effect required.

Otherwiſe, uſing the forementioned New Inſtrument; the two oppoſite Horizons are correctly brought to meet each other, and the Arch of their Diſtance paſſing through the Zenith is ſhewn on the Limb of the Inſtrument; half the Difference between which and 180 Degrees is the Effect as before.

This Method will be uſeful likewiſe on Land to know the Effect of diſtant Hills, when no real nor artificial Horizon can be had.

Problem XX. Having an Ephemeris ſhewing the Central Diſtances of the Sun and Moon, or Moon and Zodiacal Stars, to determine ſomewhat near the Hour at the Place where the Ephemeris was made for, by an Obſervation of the Diſtance of the Sun and Moon's neareſt Limb, or either the Diſtance of the neareſt or remote Limb of the Moon from a Zodiacal Star.

Rule. Caſe 1ſt. To the neareſt Diſtance of the Sun and Moon's Limbs add 32 Minutes, or half a Degree may be ſufficient, and amongſt the Diſtances of Sun

and Moon or Moon and Stars in the Ephemeris, take out the neareſt Hour for that required.

Caſe 2d. When a Star and the Moon's neareſt Limb are obſerved, add only 15 or 16 Minutes.

Caſe 3d. When a Star and the Moon's remote Limb are obſerved, ſubtract only 15 or 16 Minutes.

This doth not give the true Diſtance of Centres, becauſe that will often be affected 15, 20, or more Minutes of a Degree after this Allowance; however it may be uſeful for determining ſeveral Requiſites although the Error is greater, as will appear farther on.

Problem XXI. To determine what are the principal Data or things given, in order to find the Longitude at Sea by the Sun, Moon and Zodiacal Stars, beſides the Obſervation and Calculation.

1ſt. The true Declination of the Sun. 2d. The Sun's true Semidiameter. 3d. The Moon's true horizontal Semidiameter. 4th. The Moon's true horizontal Parallax, 5th. The true central Diſtance of Sun and Moon, or Moon and Zodiacal Stars at the End of ſhort Intervals of Time for a given Place. 6th. Correct Tables of Refraction. 7th. The obſerved Diſtance and Altitudes of the Sun and Moon, or of the Moon and Zodiacal Star, at the Place of Obſervation.

Problem XXII. Having an Ephemeris, at Sea, to determine the Sun's Declination, the Moon's true Semidiameter and her true horizontal Parallax, at the Time when her Diſtance from the Sun is taken, or from a Zodiacal Star.

Rule. Firſt get the eſtimated Diſtance of Centres by Problem 20. with which, enter the Ephemeris amongſt the Diſtances, and take out the neareſt Hour; with this Hour, enter the Table of Sun's Declination, alſo of horizontal Semidiameter and horizontal Parallax, and either add to or ſubtract from them ſuch a proportional Part as the Hour before or after Noon requires; and the true Declination, or horizontal Semidiameter, or

or horizontal Parallax will be had, which were required.

In this Method, the Truth will often be found without ſenſible Error, but if any farther Correction be wanted, the Method is ſhewn farther on.

Problem XXIII. Having the Longitude ſomewhat near the Truth by the Ship's Reckoning, and the Hour of the Day or Night at the Ship; to find the Sun's true Declination, the Moons true horizontal Semidiameter, and her true horizontal Parallax, by help of an Ephemeris, as before.

Rule. By the Ship's ſuppoſed Longitude and the Hour at the Ship, eſtimate the Hour at that Time for the Place which the Ephemeris was made for; with that Hour, proceed as in the laſt Problem, and the things will be found which were required.

Problem XXIV. Having the ſame things given as in the former Problems, to find the Moon's true Declination.

Rule. Find the Time for that Place the Ephemeris was made for, as nearly as poſſible, by the Method before mentioned, then enter the Ephemeris amongſt the daily Declinations of the Moon, and either add or ſubtract ſuch a proportional Part, as the Change of Declination to the given Hour requires; and it gives the Declination required.

Problem XXV. Having the Sun's horizontal Parallax, either 8 Seconds, 8½ Seconds, or 9 Seconds of a Degree; to find his Parallax in Altitude, at any given Elevation.

Rule. As Radius to the Coſine of the Altitude; ſo is the horizontal Parallax, to the Parallax in Altitude. This forms the Table of the Sun's Parallax in Altitude.

Problem XXVI. Having the Moon's horizontal Parallax, to find her Parallax in Altitude at a given Altitude.

Rule. Reduce the horizontal Parallax to Seconds of a Degree; and ſay, as Radius to the Coſine of the given

Altitude, ſo is the horizontal Parallax in Seconds, to the Parallax in Altitude in Seconds.

Example. Required the Parallax in Altitude for the horizontal Parallax 58m. 15s. and Altitude 32d. 30m?

Here the horizontal Parallax makes 3495 Seconds; and, as Radius to Coſine 32d. 30m. ſo 3495 to 2947 Seconds or 49m. 7s. the Parallax in Altitude required.

By this Method, the Table of parallax in Altitude is formed.

Problem XXVII. Having the horizontal Parallax and Altitude as in the former Problem, to find how many Seconds of a Degree are to be added to the Moon's horizontal Semidiameter, for her being elevated above the Horizon.

Rule. Take a 60th Part of the Moon's horizontal Semidiameter in Seconds; and ſay, as Radius to the Sine of the Altitude, ſo is that 60th Part to the number of Seconds to be added to the horizontal Semidiameter, to have her true Semidiameter at that Altitude.

Example. If the horizontal Semidiameter is 15m. 40s. and the Altitude 52d. 50m. what is the true Semidiameter at that Altitude?

Here the 60th Part of the horizontal Semidiameter is 16 Seconds nearly; and, as Radius to Sine of 52d. 50m. ſo 16 Seconds to 13 Seconds nearly, which added to the horizontal Semidiameter gives 15m. 53s. the true Semidiameter at that Altitude.

Which may be demonſtrated thus. See Figure to Art. 52. of the Introduction. When the Moon is at *b* ſhe is nearly a 60th Part of her Diſtance nearer to the Earth, than when at *d*; therefore, her Semidiameter meaſures as much more. But, ſuppoſing that Difference to ariſe from the Difference of the Diſtances *c d* and *c b*, and *e* the Moon between the Zenith and Horizon, in the Triangle *a e f* whilſt *a e* and *a f* remain invariable, *e f* will be directly as the Sine of the Angle *d a e*, which is the Altitude.

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By this Method, the Table of Parts for the Moon's Altitude, is formed.

Problem XXVIII. To find the Effect of Refraction in Altitude the Addition for Semidiameter, alſo the Parallax in Altitude, both of Sun and Moon.

Rule. Firſt find the Parallax in Altitude, and the Addition for Semidiameter as in the former Problems; and taking the Difference between the Refraction and the Parallax in Altitude, it ſhews how much the Moon is to be elevated above her viſible Place, to raiſe her to her true Place in the Heavens; and how much the Sun is to be depreſſed below his viſible Place to bring him to his true Place.

Example. If the Moon's horizontal Parallax and horizontal Semidiameter be as in the former Problem, and the Sun's Altitude, 24d. 15m. required thoſe Effects on both?

The true Semidiameter is found as before 15m. 53s. and the Parallax in Altitude 49m. 7s. or 2947 Seconds, from which take the Refraction 1m. 30s. or 90 Seconds, remains 2857 Seconds or 42m. 37s. the joint Effect for the Moon.

The Sun's Parallax in Altitude is 8 Seconds nearly, which taken from his Refraction 2m. 8s. gives 2m. or 120 Seconds the joint Effect for the Sun.

Problem XXIX. At Sea, having the angular Diſtance of the Sun and Moon, or Moon and a Zodiacal Star, taken in the Day or Night, an Ephemeris, and Watch to keep Part of a Day truly, to find the Latitude by the Moon's Meridian Altitude.

1ſt. In this Problem, the obſerved central Diſtance of the Sun and Moon or Moon and Star, with the Help of an Ephemeris, gives ſomewhat near the Hour of Obſervation for the Place the Ephemeris was made for; by which eſtimated Hour, the Watch being ſet, and let go on, 'till the Moon's Meridian Altitude is taken, the Watch will ſhew what Hour it is at the forementioned Place, and for that Hour in the Ephemeris, the Moon's true Declination muſt be taken out, in order to be ap-

plied

plied in the ſame Manner as the Declination of the Sun or a Star.

2d. The Moon's true Altitude muſt be found, by finding her Parallax in Altitude, the Difference between which and her Refraction in Altitude is additive, becauſe the Parallax in Altitude is greater than the Refraction; and if the lower Limb is obſerved, her true Semidiameter at that Altitude is additive, otherwiſe ſubductive; this gives the true Altitude of the Moon's Centre.

3d. Having the true Declination and the true Altitude of the Centre cleared from Refraction and Parallax in Altitude; the Sum or Difference is to be taken (in the ſame Manner as the Sun and Stars are uſed) to get the Meridian Altitude of the Equator, whoſe Complement is the Latitude required.

In this Problem, it is ſuppoſed that the Ephemeris predicts the Moon's Declination, without erring a Minute of a Degree; and that in the Night, the Horizon is a little illuminated by the Moon's Light, and unobſcured by Vapours. If the Moon happens to be very near either of the Tropics, the Declination will alter but a little more than a Minute in 24 Hours, in which Caſe, the Hour of Obſervation will not be wanted, nor the Watch to keep Time. But when the Declination is ſmall, it may alter 6 Degrees in 24 Hours, or 15 Minutes an Hour; and therefore, at ſuch Times, it will be difficult to determine the Latitude, to a Minute of a Degree, by not knowing the Hour at the Place for which the Ephemeris was made, without an Error of 4 Minutes of Time, that is the Longitude of the Ship within a Degree of Longitude. Therefore, in ſuch Obſervations, the Longitude muſt be known before the Latitude can be found by the Moon's Meridian Altitude. If the Hour be inferred by the Time at the Ship and the Longitude by Reckoning, the Error will not admit of a Correction, but it may by getting the true Diſtance of Centres, and aſcertaining the Longitude by the Moon, or a Variation Chart, or

any

any other Method ſufficiently correct for that Purpoſe.

Problem XXX. Having each of the three Sides of a ſpherical Triangle, to find the three Angles.

Rule. Add together the three given Sides, and from their half Sum ſubtract the Side oppoſite to the Angle ſought to get a Remainder. Then add together, the Complements Arithmeticial of the two Sides, containing the Angle ſought, the Sine of the half Sum, and the Sine of the Remainder; half the Sum of theſe four Logarithms being found in Sines, and the correſpondent Coſine Arch doubled, is the Angle ſought.

By this Theorem, the Azimuth Angle is commonly computed, having the Co-latitude, the Co-altitude, and the Polar Diſtance; alſo, the Hour Angle from Noon, by having the Co-latitude, the Polar Diſtance, and the Co-altitude.

Other Theorems, the moſt uſual ones for plain and ſpherical Calculations, are inſerted in a manner very eaſily to be underſtood, in my Treatiſe of *Practical Aſtronomy*.

Problem XXXI. To take the three cotemporary Obſervations, in finding the Longitude at ſea by the Moon.

Rule. Let three Obſervers be employed in this Obſervation; the firſt and moſt ſkilful, with the Inſtrument, in taking the Diſtance of Sun and Moon's Limbs, or Limb of the Moon and Star; the ſecond, in taking the Sun's Altitude; and the third, the Moon's Altitude at the ſame Time. Two or three ſuch Obſervations ſhould be made one after another as quick as poſſible, and the Medium of the reſpective ones, will be the reſpective Obſervations required.

The Limits of the Errors in taking the Diſtances, have been conſidered at large in the Introduction prefixed to this Work.

Problem XXXII. Having the three cotemporary Obſervations, to find the Angle at the Sun or Star, and alſo the Angle at the Moon.

Rule.

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Rule. Having taken the Sun or Star's Altitude, the Moon's Altitude, and the Diſtance of the Limbs, or Diſtance of their Centres; get the two Co-altitudes, which with the Diſtance make three Sides of an oblique angled ſpherical Triangle. In this Triangle, the Co-altitude of the Sun and the Diſtance, include the Angle at the Sun; the Co-altitude of the Moon and the Diſtance, include the Angle at the Moon. Theſe two Angles being found by the foregoing ſpherical Theorem, are the Angles required.

Problem XXXIII. Having the three cotemporary Obſervations; to find the true Diſtance of the Centres of Sun and Moon, or Moon or Star.

Rule. Firſt compute the Angle at the Sun and the Angle at the Moon. Secondly, find how many Seconds of a Degree the Sun is to be depreſſed if it be a Day Obſervation, or the Star, if it be in the Night; alſo find how many Seconds of a Degree the Moon is to be raiſed; each of theſe, by the joint Effect of Refraction and Parallax in Altitude. Thirdly, find how many Seconds are to be either added to or ſubtracted from the obſerved Diſtance, for the Sun and for the Moon, which being added or ſubtracted, gives the true Diſtance of Centres required.

The particulars concerning the making of theſe Corrections, are fully treated of, from Page 27 to 37 in the Introduction, to this Work.

Example I. Suppoſe the obſerved Altitude of the Sun's Centre cleared from Dip and Semidiameter 24d. 40m. the obſerved Altitude of the Moon's Centre cleared in the ſame manner 12d. 28m. the obſerved Diſtance of Centres 51d. 28m. 36s. and the horizontal Parallax 56m. 14s. Required the true Diſtance of the Centres cleared from Refraction and Parallax?

Solution. The Sun's Co-altitude is 65d. 14m. the Diſtance 51d. 29m. and the Moon's Co-altitude 77d. 32m.; the half Sum of the three is 97d. 7m. and Remainder 19d. 33m.; the half Sum of the four Logarithms is 9.8352 and double Coſine Arch 93d. 40m. for

for the Angle at the Sun. The Moon's Co-altitude is 77d. 32m. the Diſtance 51d. 29m. and the Sun's Co-altitude 65d. 14m. the half Sum of the three is 97d. 7m. and Remainder 31d. 53m.; the half Sum of the four Logarithms is 9.9182, and double Coſine Arch 68d. 10m. for the Angle at the Moon. The Sun's Parallax in Altitude is 8 Seconds, which taken from the Refraction for the Sun gives 115 Seconds for the Hypothenuſe of the little Triangle at the Sun. The Moon's Refraction 252 Seconds taken from the Moon's Parallax in Altitude 3293 Seconds, gives 3041 Seconds for the Hypothenuſe of the little Triangle at the Moon. As Radius to Coſine of the Sun's Angle 93d. 40m. or rather it's Supplement 86d. 20m. ſo 115 Seconds to 7 Seconds, which are additive becauſe the Sun's Angle is greater than 90d. otherwiſe they would have been ſubductive. As Radius to Coſine of the Moon's Angle 68d. 10m. ſo 3041 Seconds to 1131 Seconds, which are ſubductive becauſe the Moon's Angle is leſs than 90d. otherwiſe they would have been additive. From 1131 take 7 remains 1124 Seconds or 0d. 18m. 44s. which taken from 51d. 28m. 36s. gives 51d. 9m. 52s. the true Diſtance of Centres required.

Example 2d. Suppoſe the Altitude of a Star 15d. 27m., the Altitude of the Moon's Centre 27d. 35m. both being cleared from Dip; the obſerved Diſtance of the Star and Moon's Centre 102d. 30m. 5s. and the horizontal Parallax 57m. 4s. Required the true Diſtance of the Centres?

Solution. The Stars Co-altitude is 74d. 33m. the Diſtance 102d. 30m. and the Moon's Co-altitude 62d. 25m.; the half Sum of the three is 119d. 44m. and Remainder 57d. 19m.; the half Sum of the four Logarithms 9.9451, and double Coſine Arch 56d. 18m. for the Angle at the Star. The Moon's Co-altitude is 62d. 25m. the Diſtance 102d. 30m. 5s. and the Star's Co-altitude 74d. 33m.; the half Sum of the three is 119d 44m. and Remainder 45d. 11m; the

half Sum of the four Logarithms is 9 9262, and double Coſine Arch 64d. 56m for the Angle at the Moon. The Star's Parallax in Altitude is 0 Seconds, which taken from the Refraction for the Star gives 204 Seconds for the Hypothenuſe of the little Triangle at the Star. The Moon's Refraction 109 Seconds taken from the Moon's Parallax in Altitude 3035 Seconds, gives 2926 Seconds for the Hypothenuſe of the little Triangle at the Moon. As Radius to Coſine of the Star's Angle 56d. 18m. ſo 204 Seconds to 113 Seconds, which are additive becauſe the Star's Angle (like that of the Sun) is leſs than 90d. As Radius to Coſine of the Moon's Angle 64d 56m ſo 2926 Seconds to 1240 Seconds, which are ſubductive becauſe the Moon's Angle is leſs than 90d. From 1240 take 113 gives 1127 Seconds or 18m. 47s. ſubductive from 102d. 30m. 5s. ſo the true Diſtance of Centres is 102d. 11m. 18s. which was required.

Example 3d. Suppoſe the Star's Altitude obſerved 48d. 23m., the Altitude of the Moon's Centre cleared from Dip and Semidiameter 64d. 32m. the obſerved Diſtance of their Centres 33d. 15m. 20s, and the horizontal Parallax 55m. 30s; to find the true Diſtance of the Centres?

Solution. The Star's Co-altitude is 41d. 37m.; the Diſtance 33d. 15m. and the Moon's Co-altitude 25d. 28m.; the half Sum of the three is 50d. 10m. and Remainder 24d. 42m.; the half Sum of the four Logarithms is 9.19725, and double Coſine Arch 40d. 20m. for the Angle at the Star. The Moon's Co-altitude is 25d 28m.; the Diſtance 33d. 15m. and the Star's Co-altitude 41d. 37m.; the half Sum of the three is 50d. 10m. and Remainder 8d. 33m.; the half Sum of the four Logarithms is 9.84425, and double Coſine Arch 91d. 48m. for the Angle at the Moon. The Star's Refraction is 50 Seconds, for the Hypothenuſe of the little Triangle at the Star. The Moon's Refraction 27 Seconds taken from the Moon's Parallax in Altitude 3330 Seconds, gives 3006 Seconds for

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the Hypothenuse of the little Triangle at the Moon: As Radius to Coſine of the Star's Angle 40d. 20m. ſo 50 Seconds to 38 Seconds, which are additive becauſe the Star's Angle is leſs than 90d. As Radius to Coſine of the Moon's Angle 91d. 48m. or 88d. 12m. ſo 3006 Seconds to 94 Seconds, which are additive becauſe the Moon's Angle is more than 90d. But here on account of the ſhortneſs of the Diſtance, and the Moon's Angle being near a Right Angle, the Diſtance from the Star to the Moon, is longer than from the Star to the foot of the little Perpendicular 46 Seconds, by which the 94 Seconds additive for the Moon is too much; therefore 48 Seconds and 38 Seconds 1m. 26s. added to 33d. 15m. 20s. gives 33d. 16m. 46s. the true Diſtance of Centres required.

Example 4th. Suppoſe the Star's Altitude 53d. 15m. the Moon's Altitude 64d. 39m. the obſerved Diſtance of Centres 56d. 17m. 38s. and the horizontal Parallax 61m. 10s. Required the true Diſtance of Centres?

Solution. Here, the Stars Angle is 22d. 44m. 1. the Moon's Angle 32d. 40m.; the Star's Refraction 42 Seconds; the Moon's Refraction 27 Seconds; the Moon's Parallax in Altitude 1571 Seconds; the Hypothenuſe of the Star's little Triangle 42 Seconds, and that of the Moon's 1544 Seconds; the additive Seconds for the Star's little Triangle 39; the ſubductive Seconds for the Moon's 1300; their Difference 21m. 1s. ſubductive from 56d. 17m. 38s. and 55d. 56m. 37s. the true Diſtance Centres required.

91ſt. From the foregoing Examples and their Solutions, and others that might be introduced, it appears, that this Method is eaſy and certain, requiring no other original Principles than what the Navigator either is or ought to be acquainted with, nor any other Tables than are or ought to be in every Ship at Sea. Theſe are Circumſtances which ingenious Perſons will ſet ſome value on, ſeeing the Problem is not of the moſt compendious and eaſy kind, and it hath not been

may observe that there have been given various Methods of solving it, which are not only tedious, but can be traced from their Originals only by expert Mathematicians and compleat Philosophers. On the contrary, Tables which are ingeniously adapted and calculated for solving this Problem in an easy and correct manner, cannot but be of singular Use to such as choose to decline entering into the Reasons or first Principles on which the Computations are founded, and these may also be readily applied by others, many times with more Certainty than a pure analysis of the Problem admits of, through Imperfection in the Computor; whenever such Tables are wanting, there is no other way of supplying their Use but by some original Method, one of which hath been before mentioned. The Observer will therefore do well, to consider the Solution of this Problem as much from the original Principles on which it depends as he is able, but in general to practise such a Method as is most easy and at the same Time correct enough for the Purpose. For, being enabled to solve the Question in a general way, he will make such allowance for Refraction within a few Degrees of the Horizon, as the State of the Climate or Season requires, and thereby expunge those Errors which arise through a supposed constant State of the Atmosphere.

Whilst the Solution is sought by an Original way, it may be had by finding the Sides or Angles of the spherical Triangles which lead to the Answer, but these will be correctly computed only by logarithmic Tables shewing the correspondent Arches to Seconds of a Degree, and such are not to be had; otherwise, perhaps one of the most easy Methods of Solution would arise from the whole spherical Triangle, as it is formed in the Sphere. For Instance, in the Triangle *edg* if the Angle *e* be found, and by it with the two Sides *ge* and *eg* the Side *fg* is found, it is the Side required. Thus is the most natural and at the same time correct Method of Solution that can be wished

for,

for, but it requires the Angle to a Second of a Degree; and that the logarithms taken out for the containing Sides, be to Seconds of a Degree likewise, otherwise the required Side will not be found to that Accuracy. It is also necessary (whatever Method of Solution be applied) to consider how the Data are to be best prepared for Calculation. For Instance, if the Sun or Moon be near the Horizon, the Refraction for the Limb must not be taken for that of the Centre, but a correction made previous to the Solution by any of these Methods. If the Requisite Tables be applied, the Solution will come out very easily by the Formula which was drawn up by me soon after those Tables were published.

93d. The true Distance of the Centres of Sun and Moon or of Moon and Star being found, and compared with the assumed Distance for which the Moon's Semidiameter and horizontal Parallax were taken out, it will be seen whether there be any Error arising from that Assumption, and if the Error be enough to affect the Longitude, it should be corrected. For Instance, if on April 23. 1780, the observed Distance of Sun and Moon's Centres be 48d. 12m. 10s. and the true Distance of those Centres 48d. 32m. 20s. the supposed Time at the Place where the Ephemeris was made, will be two thirds of an Hour different from Truth; the Moon's Semidiameter will be correctly taken out by it, but the horizontal Parallax taken out by the same Hour will be erroneous by a Second and half; and the Sun's Declination will not be erroneous a Minute of a Degree; in many other Cases, the Error in each will be less, and these produce but a trifling Effect on the true Distance of the Centres in comparison with other Errors. But all this is most certainly corrected by replacing the true Distance instead of the assumed, and taking out the Requisites accordingly.

94th. The Error of Longitude to which Ships are frequently subject by Reckoning, is not so little as 5 or 6 Degrees, when they have been on a long Voyage;

if it

if it amounted to 10 Degrees, and the Reckoning together with the Hour at the Ship were applied for taking out these Requisites, no greater Errors could arise than the forementioned; and this seems to be the Reason why they are commonly applied, although there be no particular necessity for them, but if the Longitude be lost, or there be no dependance on the Reckoning, the Assumption will discover the Truth. Otherwise, the Longitude may be found, where it is practicable, by the fourth Method, the Variation of the Magnetic Needle, and that with the Hour at the Ship will be sufficient for getting the Requisites as correctly as they are wanted. In that Application, the Variation Chart may be considered as no small Addition to the Method of finding the Longitude by the Moon, as it will readily shew the Longitude in a multitude of Cases, within 2 Degrees, and the Ship's Clock may often be true to 4 or 5 Minutes of Time, and from both of these, the Time for the Place the Ephemeris was made for, may be readily inferred within 15 Minutes of Time, and half that Accuracy will be sufficient for the Requisites before mentioned.

Problem XXXIV. To take the Distance of Sun and Moon's Limbs, or the Moon's Limb and a Zodiacal Star, as accurately as possible.

Solution. This most essential part of the Observation, is performed by taking those Distances, as nearly as possible at the end of small and equal Intervals of Time. On Land (where no Impediment ariseth through the Motion of a Ship) one of these Observations may be made and written down in a Minute of Time, by a Person who can use the Instrument in the best Manner, and the Medium of three such Observations, will be sufficiently near the Truth. But at Sea, it may sometimes require double that Interval to get three Observations or 6 Minutes of Time, and if 8 or 10 Minutes be wanted to make 4 or 5 Observations, the Medium may be sufficiently correct for the Longitude. The respective Mediums of the Sun's Altitude

and

and Moon's Altitude, will give the other of the contemporary Observation's, with as much Accuracy as is wanted. In this Practice, one Distance observed may differ from another more than is expected, by perhaps one or two Minutes of a Degree; this should be no Discouragement to the Observer, nor occasion him either to reject or slight such differing Observations. The Truth itself cannot be discovered, but it is probable the Medium may differ therefrom, by no great quantity (as mentioned in the Section of *Combination of Errors*, in the Introduction.) An Error of 2 or 3 Minutes in the Distance of the Limbs, may, thus happen to counteract two or even four Errors of 15 Seconds of a contrary kind, and thereby greatly promote the Truth. Injudicious Practices of this kind seem to have been of Prejudice to this Method, and if not carefully avoided may set aside the Merit which is due to it. Many Years since, when this Method was first practised, an ingenious Astronomer abroad made several Observations at Sea, and computing them singly, found a disagreement of 2 or 3 Degrees in Longitude; if those Errors were all plus or all minus, the Tables, the Instruments or the Observations might be imperfect, and these being corrected, the Method was good; if they were some plus others minus, the Truth might be nearly approached by the one counteracting the other. But, on these Circumstances, the Method itself was illjudged of, both by the ingenious Observer and others who were good Astronomers.

Problem XXXV. Having the true Distance of Centres of the Sun and Moon, or Moon and Zodiacal Star, and an Ephemeris, shewing either the hourly, two hourly, 3 hourly, 4 hourly, or 6 hourly Distances of the Sun and Moon, or Moon and Zodiacal Stars; to find the Solar Time at the Place for which the Ephemeris was made.

In this Problem, the apparent Motion of the Moon through the Heavens (whereby she appears to separate from the Sun and Zodiacal Star's from West toward

East,

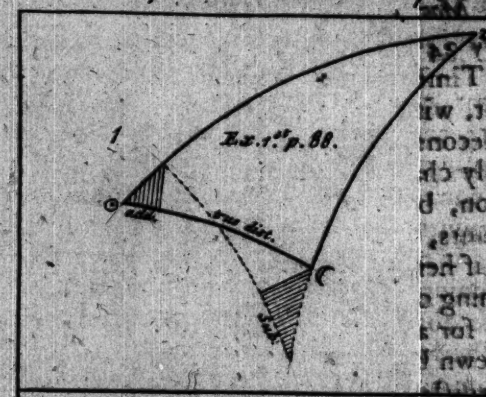
East, at the end of every 24 Hours) is unequal, in the ſmalleſt Intervals of Time; firſt, becauſe ſhe is always moving in her Orbit, with either an accelerated or retarded Motion; and ſecondly, becauſe the Earth in it's Orbit, is continually changeing it's Poſition to and Diſtance from the Moon, beſides an Inequality of Motion. On theſe Accounts, but chiefly that of the Moon's unequal Motion, if her Place in the Heavens be predicted for the Beginning of every Hour throughout the Year, her Place for any intermediate Time will be more accurately ſhewn by a proportional Part, than if her Place be predicted for the Beginning of every three Hours, from the ſame Aſtronomical Tables, and they are ſuppoſedly true. By the ſame reaſoning, Predictions for the Beginning of every three Hours are more exact, for inferring intermediate proportional Parts, than thoſe for the Beginning of every ſix Hours. Alſo 6 hourly ones are better than thoſe of 12 Hours; and 12 hourly ones are better than the daily Place of the Moon in the Heavens, or her Diſtance from the Zodiacal Stars.

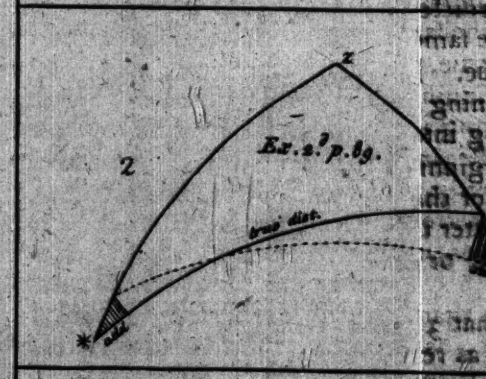
Hence it is concluded, that 3 hourly Predictions are ſufficient for this Purpoſe, as retaining no greater Errors by proportioning for intermediate Time, than will be of inconſiderable Effect in applying the Obſervation and Calculation to the Longitude at Sea; and therefore theſe are the Times to which thoſe Diſtances are computed and publiſhed in the Ephemerides or daily Tables.

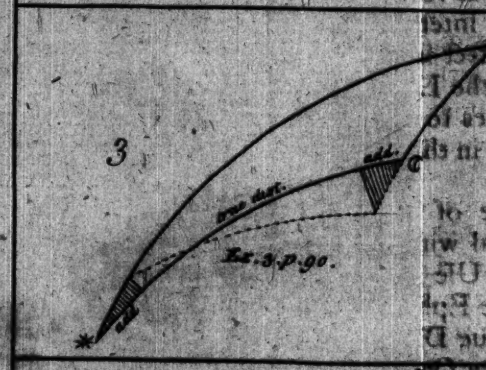
When the true Diſtance of the Centres is found, and the Obſerver is provided with an Ephemeris computed in this manner, it's Uſe is to be determine the Solar Time at the Place the Ephemeris was made for, at that Inſtant when the true Diſtance of Centres was taken. Here the ſucceſſive Order of Time, and the additive Hours, Minutes and Seconds of Solar Time to the firſt of the three hourly Times is the only thing wanted, whether the Diſtances be increaſing or decreaſing; hence a general Method of finding this additive quantity

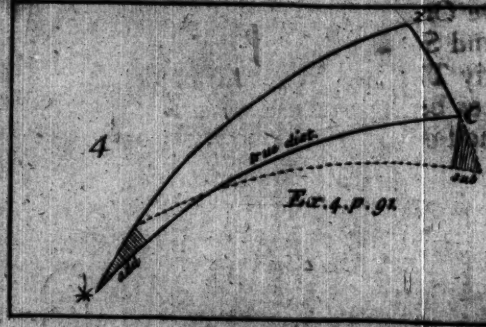
Cases of true Distance of Centres.

Plate 9.

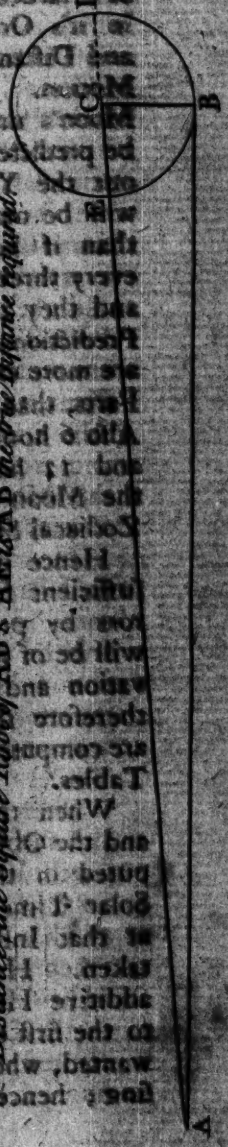








In short Distances, if C be the Mids Perpendicular, the Distance the Square Root of AD × AE + AB² be equal



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quantity of Time, by a ſingle Proportionality is as follows.

Solution. Subtract that three hourly Diſtance which is leſs than the true Diſtance of Centres from that which is the next greater, and make the Remainder the firſt Term in the Rule of Three direct; make 3 Hours the ſecond Term; take the Difference between the firſt three hourly Diſtance and the true Diſtance of Centres, and make it the third Term; then, reduce the firſt and third Terms, and multiply and divide, as in the Rule of Three, and the fourth Term is Time, which being added to the firſt of the two three hourly Times, gives the Time at the Place for which the Ephemeris was made.

Example 1. Suppoſe the firſt three hourly Diſtance 68d. 30m. 11s. at 12 Hours; the ſecond 70d. 8m. 7s. at 15 Hours; and the true Diſtance of Centres 69d. 24m. 15s. Here, as 1d. 37m. 56s. to 3 Hours; ſo 0d. 54m. 4s. to 1h. 39m. 22s. which added to 12h. gives 13h. 39m. 22s. the Time required.

Example 2. Suppoſe the firſt three hourly Diſtance 79d. 14m. 27s. at Noon; the ſecond 77d. 42m. 15s. at 3 Hours; and the true Diſtance of Centres 78d. 32m. 20s. Here, as 1d. 32m. 12s. to 3 Hours; ſo 0d. 42m. 7s. to 1h. 22m. 13s. which added to Noon, gives the time required.

Problem XXXVI. Having the Latitude of the Place, and the three cotemporary Obſervations, taken in the Day-time, with the Uſe of an Ephemeris; to find the Longitude.

Solution. This Problem contains the application of ſeveral of the former. Firſt, by having the Latitude Declination and Altitude of the Sun, the Solar Time at the Ship is to be found. Secondly, by the three cotemporary Obſervations, and the Uſe of the Ephemeris, the Solar Time is found for the Place where the Ephemeris was made. Thirdly, the Difference of theſe Times turned into Degrees, by allowing 15 Degrees to an Hour of Time, gives the Longitude re-

quired, East when the Time at the Ship is greateſt, but otherwiſe Weſt, as in the Method by a Watch.

Problem XXXVII. Having the Latitude, and the three cotemporary Obſervations taken in the Night, with the Uſe of an Ephemeris; to find the Longitude.

Solution. The Time at the Ship muſt be found by having the Latitude, the Star's Declination and Altitude, as is before directed in the Problem for that Purpoſe; and the Time for the Place the Ephemeris was made for, muſt be found in the ſame manner as is directed for the Sun; then the Difference of theſe two Times is the Longitude required.

In the Practice of this Problem, the ſame Caution ſhould be uſed, not to obſerve when the Star is too near the Meridian, as is directed for the Sun; and if the Horizon is not well enough defined for making the Obſervation, the Star's Altitude may be taken by Reflection from either a whirling Speculum, a floating Mirror, or by ſeveral other Methods, when it is of the firſt Magnitude, and the Weather is favourable, but it oftens happens, that this cannot be practiſed with Accuracy; and therefore, it is readier to ſet a good Watch to Solar Time, by an Obſervation of the Sun, and let it go on and ſhew the Time at the Ship when the Obſervation is made in the Night by the Stars.

It may be farther noted, that as it will be neceſſary to know the Star is not near the Meridian when the Obſervation is made, this cannot be known without a knowledge of the Variation at the Place of Obſervation; and in this Reſpect alſo that Chart will be aſſiſting in finding the Longitude by the Moon and Zodiacal Stars.

Problem XXXVIII. Having the three cotemporary Obſervations and the Ephemeris, to compute the Longitude very eaſily, by help of the Requiſites Tables.

Solution. Firſt prepare the Problem for a Solution, after

A Chart of the Zodiacal Stars Used in Fin
by the Moon. By S. Dunn, Teacher of

Published with Improvements by S. Dunn N°. 6. Clement's In

Lat.

80 70 60 50 40 30 20 10

Fomalhaut

Feb 12

Jan 28

Dec 27

Nov 27

Oct 22

Sept 22

Aquila

Capricorni

Antares

Jupiter

Libræ

Spica

Hydræ

Canobus

Sirius

Aldebaran

Procyon

Orion

Rigel

Achar

Hour Nov 29

Lat.

80 70 60 50 40 30 20 10

Find the Sun's, Moon, & Planets Places in the Zodia
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to the Left, for before Noon to the Right of t

n finding the LONGITUDE at SEA.
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Zodiac, by an Ephemeris, & their Positions to
ction. For Hours of a Day past Noon count
t of the Sun, gives the Stars on the Meridian.

Of Longitude by the Sun, Moon and Zodiacal Stars.

after the manner before directed; and then go through the Proceſs according to the Formula adapted for that Purpoſe, and it will give the Longitude required.

Problem XXXIX. To examine an Ephemeris, and diſcover the Perfection or imperfection of it's Predictions.

To perform this in a compleat manner, requires the moſt perfect Aſtronomical Inſtruments that can be had, and the moſt diligent and expert Obſerver to apply them. What chiefly belongs to the Longitude at Sea, are the Places of the Sun, Moon, and ſome of the Fixed Stars.

The Principal Inſtruments to be uſed in examining their Places, are the aſtronomical Clock regulated to beat Seconds of ſiderial Time; the Tranſit Inſtrument placed in the Plane of the Meridian; and the aſtronomical Quadrant placed in that Plane, for taking Meridian Altitudes.

The equatorial Diſtances of the Sun from the reſpective Stars, alſo the like Diſtances of the Moon from the Sun and Stars, are obſerved as correctly as poſſible, by the Time ſhewn in ſiderial Hours, Minutes and Seconds, which being reduced to Degrees, Minutes and Seconds, gives their obſerved Right Aſcenſions; by help of which and their obſerved Declinations their Longitudes and Latitudes are computed, and theſe will either agree or diſagree with the Predictions, according to the Perfection or Imperfection of the Tables from which their Places were calculated.

If there happens to be any Error in a Prediction that is diſcernible to the Senſes, an Obſervation will certainly diſcover it, provided it be made with due Care and with good Inſtruments by an able Obſerver; on which Account, aſtronomical Calculations are always liable to a ſevere Examination, and frequently Correction by Obſervations; but whenever the predicted Places of the Sun and Moon can be depended on, they may be applied inſtead of a correſpondent

Obſerver, and ſubſtituted in finding the Longitude at Sea by the Sun, Moon and Zodiacal Stars.

In this manner may the forementioned Obſervations on the Lunar Eclipſe be compared with the beſt Predictions, and they will be thus.

1. Obſerved Beginning by the naked Eye at 10h. 15m. 42s.; obſerved End by the ſame at 13h. 49m.; Middle of the two Obſervations 12h. 2m. 21s.; ſubtract 5m. 48s. to reduce this (as per Art. 69. page 63.) gives the Middle at 11h. 56m. 33s. Solar Time at Greenwich for the Middle of the Eclipſe by the naked Eye.

2. Obſerved Beginning by Refracting Teleſcope at 10h. 16m. 25s.; obſerved End by the ſame at 13h. 48m. 20s.; Middle of the two 12h. 2m. 21s.; ſubtract 5m. 48s. as before gives 11h. 56m. 33s. Solar Time at Greenwich for the Middle of the Eclipſe by the Refracting Teleſcope.

3. Obſerved Beginning by Reflecting Teleſcope at 10h. 17m. 40s.; obſerved End by the ſame at 13h. 46m. 55s.; Middle of the two Obſervations 12h. 2m. 18s.; ſubtract 5m. 48s. gives 11h. 56m. 30s. Solar Time at Greenwich by the Reflecting Teleſcope.

Hence, Middle by the naked Eye at 11h. 56m. 33s.
by the Refractor - 11h. 56m. 33s.
by the Reflector - 11h. 56m. 30s.
by the Medium - 11h. 56m. 32s.
by the Ephemeris - 11h. 56m. 20s.
Difference 12s.

From which, it appears, that the Sun and Moon's Place were then predicted accurate enough for determining the Longitude without erring more than 3 Miles at the Equinoctial, according to theſe Obſervations. But this may be more accurately determined by comparing correſpondent Obſervations of the Biſections of the Spots.

Problem XL. To correct the Tables of the Sun and Moon by help of aſtronomical Obſervations; and thereby

thereby render them more perfect for Application to the Diſcovery of the Longitude at Sea.

When an Obſervation hath been made of the Hour, Minute and Second of Siderial Time, the Sun or Moon tranſits the Meridian of a Place for which an Ephemeris is made, and it being reduced to Solar Time, agrees with the Prediction, it may be ſuppoſed, that the Tables from which the Prediction was made are correct, but otherwiſe that they are imperfect and want Correction; and when they are corrected, Predictions made from them ſhould ever after agree with Obſervations.

There are ſeveral Methods by which ſuch Corrections may be made, but one only hath a Tendency toward the making them perfect, and that muſt be aſſiſted by the moſt accurate Obſervations and the beſt Theories.

If the Earth moved round the Sun in a Circle, and equal Spaces in equal Times; alſo, if the Moon moved round the Earth in a Circle, equal Spaces in equal Times, there would be but little Difficulty in aſſigning their Places in the Heavens for any Time to come, but whilſt they move by the Law of Gravity, and the other Planets have ſome ſmall Power for diſturbing their Motions, the Taſk becomes the more difficult, and cannot be performed without due Conſideration of the Forces which give Motion to the Earth and Moon in their reſpective Orbits.

The firſt Step to be taken in order to correct the Tables, is to transfer the geocentric to the heliocentric Place; that is, to conſider how the Earth and Moon would appear ſituated at the Time of Obſervation, if an Obſerver was at the Sun, whilſt he cannot but be on the Earth; and then to make ſuch Alteration in the Tables as the Obſervation points out and requires.

For Inſtance, if the Earth be at *b*, and the Moon at *l* moving toward *k*, ſhe will be retarded in Motion reſpecting the Earth, but accelerated in Motion toward

s the Sun. On the contrary, in moving from *m* toward *i*, ſhe will be accelerated toward the Earth, but retarded by her Poſition to the Sun. Conſequently, whilſt the Moon's Perigee and Apogee are continually changing their Poſition to the Sun and Earth, a great Variety of different Powers, though ſmall, will be continually taking off that Regularity which would ſubſiſt in the Lunar Orbit, if but one Force acted on the Moon, in her Motion round the Earth.

Hence it follows, that in order to correct the Lunar Tables effectually, it is neceſſary to know either the real or relative Diſtances of the Earth from the Sun and the Moon from the Earth at all times, beſides the quantity of Matter in each, and the Effect produced on each by the other Planets. For, theſe only can furniſh ſufficient Data to correct the Lunar Orbit, and make the Predictions agree with Obſervations. In any other Hypotheſis of Correction, the Errors may return, if an Alteration be made in the whole Orbit which ought to have been diſtributed through a Part only, or if it be not clearly foreſeen what are the Cauſes producing the Errors, and exact Allowances made for them when and where they are required.

For theſe Reaſons, the Interval of 18 Years 10 days 7 hours, is thought a proper Time to obſerve the Moon's Place in the Heavens, becauſe in that Interval, her Orbit makes all its various Poſitions to the Sun and Earth, except ſome ſmall Difference ariſing through other Cauſes in the Solar Syſtem; and therefore, a compleat Courſe of Obſervations, from Day to Day, throughout that time, would be ſufficient to correct the principal Errors in the Lunar Tables, if they could be made, but for want of ſuch, and to make the intermediate Parts compleat, the Theory together with correct Obſervations, are to be applied in the manner before mentioned.

By ſuch a Correction, the Earth's Place (or apparently the Sun's Place) will agree with predicted Diſtances from the Zodiacal Stars for all times to come. The

Moon's

A New and Easy FORMULA for the LONGIT

Central Dist. — Time at Greenw.

☉ & ☽'s Lambs observed		
☉'s true Semidiameter		
☽'s true Semidiameter		
Parts for ☽'s Altitude		
Observed dist. of Centres		= L
Least Alt. Centre clear		
Greatr Alt. Centre clear		
Parts by Top & Side Tab.	1	2
Dist. Centres	Co. ar.	
A =	Common Logar.m	
B =	Parts in Tab. II. add if the Dist. is above 90.d	
=		
add L =		
D =		

Hor.l Par.x	Propor.l Log.m
☉'s Altitude	Co. Secant
Number D.	Sine
1.st Arc	Propor.l Log.m

Hor.l Par.x	Propor.l Log.m
☽'s Altitude	Co Secant
Number D.	Tangent
2.d Arc	Propor.l Log.m

C =	Add the Arcs if D. is above 90.d
D =	Add C to D if 1.st Arc is least & D. is under 90.d
E =	
F =	Last Corr.n add if D. is under 90.d
P =	True Dist. of Centres.

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			2)		
Arc			Co. Sine		
15)	2	(	H ′ ″		
					à Noon
	H. ′ ″				
Time at the Ship					
Time at Greenw.r					
				° ′	
Ship's Longitude			or		

The Co. ar. Sine, Tangent, &c. of Degrees when more than 90 is that of the Supplement to 180. The Co. ar. is Co. Sec. less 10 in the Index. In this Formula, where Addition is not to be made subtraction must. For correct Time, at the Ship, see my New Epitome of Navigation.

d. Nov.r 19th 1777 by Samuel Dunn, Minden Lane, Covent Garden, Turnmaker.

Of Longitude by the Sun, Moon and Zodiacal Stars.

Moon's Apogee and Perigee; her aſcending and deſcending Nodes, where ſhe croſſeth the Earth's Orbit; her accelerated and retarded Motions; her Longitude and Latitude and other Particulars, with her true central Diſtance from the Sun and Zodiacal Stars, may be predicted and the Computations found to agree very nearly with Obſervations.

XXVII. *Of Longitude by correct Variation Charts.*

95th. The Nature and Uſe of Variation Charts of the Magnetic Needle, hath been ſo fully explained from page 41 to 56 of this Treatiſe, that there is leſs Occaſion for entering into Particulars concerning them here.

In the three former Methods of finding the Longitude at Sea, but little if any thing can be done when the Latitude is not known near the Truth. In that by the uſual Method of Sailing, the Correction of the Longitude depends on the obſerved Latitude. In the Method by Help of a Watch, the Time at the Ship depends on the obſerved Latitude, and ſo it doth in the Method by the Sun, Moon and Stars. In all three of theſe Methods the Latitude ſhould be known within 10 or 12 Minutes of a Degree, in ſome Caſes leſs, otherwiſe the Errors may be conſiderable. But in this Method by correct Variation Charts, a leſs accuracy in the Latitude may be ſufficient in many Caſes, and ſometimes there may be no neceſſity of knowing it within many whole Degrees, where it is known that the Lines of equal Variation nearly coincide with the Meridians.

Problem I. To direct the Index of the Azimuth Compaſs toward the Sun, Moon or a Star, when they are near the Horizon.

Rule. The Index being fixed at the Beginning of the Degrees, turn it with the Frame to which it is fixed, until the Centre of the Sun, Moon or Star, is ſeen through the Sights, and it is the Direction required.

Problem II. To take the Bearing of either the North

North Point of the Needle or Card, whilſt the Index is directed toward the Sun, Moon or Star.

Rule. Whilſt one Perſon is carefully keeping the Index in that Direction, let another take the greateſt and leaſt number of Degrees pointed to by that North Point of the Needle or Card, during 2, 4, 6, or more Vibrations; and the Medium will be the Degrees and Minutes required.

Examples. Suppoſe theſe Vibrations to give 16d. and 12d. the Medium is 14d. the Anſwer. Suppoſe they give 16d. 12d. 15d 11d. the Medium is 13d. 40m. the Anſwer. Suppoſe they give 16d. 12d. 15d. 11d. 14d. 10d. the Medium is 13d. Suppoſe they give 16d. 12d. 15d. the Medium is 14d. 20m. Suppoſe they give 16d. 12d. 15d. 11d. 14d. the Medium is 13d. 36m. Theſe Mediums are 14d.; 13d. 40m.; 13d.; 14d. 20m.; 13d, 36m.; hence, the Medium of the whole is 13d. 45m.

By this Method, it will be often practicable at Sea, to take the Bearing of the North Point of the Needle to a few Minutes of a Degree.

Problem III. To keep the Index truly directed toward the Sun, when his Altitude is great or is near the Zenith.

Rule. Let the Index when fixed to it's Frame be ſo moved toward the Sun, that the Thread joining the Sights of the Index, throws it's Shadow coinciding with the Line drawn from End to End, through the Middle of the Index, and it is the Poſition required.

Problem IV. To obſerve how many Degrees and Minutes the Sun, Moon or a Star, bears either north-eaſtwardly, northweſtwardly, ſoutheaſtwardly or ſouth-weſtwardly, from the North or South Point of the Needle or Card, at the Time of Obſervation.

Rule. Direct the Index and take the Bearing of the North or South Point of the Needle, as in the former Problems; and it gives the Anſwer required.

Problem V. Having the Latitude of the Place, and the Declination of the Sun, to find the Amplitude; that

that is, at what horizontal Diſtance he riſeth from the true Eaſt, either northward or ſouthward therefrom in the Morning; and at what Diſtance he ſets from the true Weſt in the Evening.

Rule. As the Coſine of the Latitude is to the Sine of the Declination; ſo is Radius to the Sine of the Amplitude. Otherwiſe, thus. As Coſine of the Latitude is to Radius; ſo is the Sine of the Declination to the Sine of the Amplitude.

Otherwiſe thus. As Radius to Secant of the Latitude; ſo is the Sine of the Declination to the Sine of the Amplitude. Otherwiſe thus. As Radius to the Sine of the Declination; ſo is the Secant of the Latitude to the Sine of the Amplitude.

Which may be illuſtrated by Example thus.

Suppoſe the Latitude 51°. 30′ N. and the Sun's Declination 12°. 32′ N. required the true Amplitude?

By the firſt Rule. As Coſine 51d. 30m. to Sine of 12d. 32m. ſo Radius to Sine of 20d. 24m. the Amplitude.

By the ſecond Rule. As Coſine 51d. 30m. to Radius; ſo Sine of 12d. 32m. to Sine of 20d. 24m. the Amplitude.

By the third Rule. As Radius to Secant of 51d. 30m. ſo Sine of 12d. 32m. to Sine of 20d. 24m. as before.

By the fourth Rule. As Radius to the Sine of 12d. 32m. ſo Secant of 51d. 30m. to Sine of 20d. 24m. as before.

Problem VI. Having the Latitude of the Place, the Altitude of the Sun, and his Declination; to find the Azimuth or true horizontal Bearing of the Sun from the Point of the Horizon under the elevated Pole.

Rule. Add together the Co-latitude, the Co-altitude and the Polar diſtance, and from their half Sum, ſubtract the Polar Diſtance to get a Remainder. Then add together the Co. ar. of the Co-latitude; the Co. ar. of the Co-altitude; the Sine of the half Sum; and the Sine of the Remainder; and having taken the half

Sum of theſe four Logarithms, find it in Sines; take the correſpondent Coſine Arch and double it, gives the Anſwer.

Example. In Latitude 50d. 0m. N. the Sun's Declination being 12d. 20m. N. and the Altitude of the Sun's Centre cleared from Dip and Semidiameter 18d. 24m. before Noon. Required the Azimuth or true Bearing of the Sun at that Time?

Here, the Co-latitude is 40d. 0m; the Co-altitude 71d. 36m. and the Polar-diſtance 77d. 40m.; the half Sum of the three is 94d. 38m. and Remainder 16d. 58m.; the half Sum of the four Logarithms is 9.8392; and it's double Coſine Arch 92d. 39m. the Azimuth or Sun's Bearing at that Time eaſtwardly from the North Point of the Horizon.

Had it been Afternoon, the Bearing would have been weſtwardly from the ſame North Point.

Problem VII. Having the Latitude of the Place, the Altitude of the Sun's Centre taken before or after Noon, and the Bearing of the Sun's Centre, taken by the Compaſs at the ſame time, to determine the Variation.

Rule. Compute the true Azimuth by the former Problem, and compare it with the obſerved Azimuth, and the Variation will be found which was required.

In this Problem the principal Difficulty is in determining whether the Variation be eaſting or weſting; this is readily ſhewn by a Rectifier, ſhewing the Points of the Compaſs both for the true Horizon and the magnetic Horizon. The Sun's true Azimuth is found on the true Horizon; and the Point of the magnetic Horizon on which the Sun is obſerved is brought thereto, ſo the magnetic North Point of the Card, ſhews whether the Variation is eaſting or weſting.

The Plate at the End of *Practical Navigation*, is deſigned for this Uſe. But the moſt maſterly and general Way of doing this, is by making a ſmall Circle and croſſing it with two Lines drawn through the Centre at right Angles to each other, the North being uppermoſt

uppermoſt; then ſetting off any Degrees either by Eſtimation or by the Circle, from the true North to where the Sun is, as is found by the Problem, and from that Point where the Sun is ſo found, as many Degrees to the right or left thereof as the magnetic North is obſerved from the Sun, gives the magnetic North; which, (from the Problem) will either fall beyond or ſhort of the true North, and thereby ſhew the Variation and what kind it is of, whether eaſting or weſting, which was required.

Problem VIII. Having the Bearing of the Sun by the Compaſs, both in the Morning and Afternoon, when the Altitudes of the Sun are equal; to determine the Variation of the Compaſs.

Rule. Take the Bearing of the Sun's Centre both in the Morning and Afternoon, when the Altitudes are equal, the Middle Point between them on the Card is the true North Point of the Horizon, which being compared with the magnetic North gives the Variation required.

This Problem may be practiſed for any Time of the Morning and Afternoon, when the Sun is not far from either of the Tropics. But when he is near the Equator, it will be beſt to obſerve as near as Opportunity permits before and after Noon, provided it be about an Hour from the Meridian Tranſit; for then, the Altitude may be taken ſufficiently exact, and there can be no great Error by the Change of the Sun's Declination in the Interval of Time between the two Obſervations.

Problem IX. To determine when the Sun's Centre will be in the true Horizon of the Sea, ſuppoſing it affected by Refraction in Altitude only.

Rule. Firſt determine, what is the horizontal Refraction for the Place of Obſervation, and ſo much muſt the Sun's Centre be elevated above the Horizon the Inſtant when that Centre is in the true Horizon.

Example. Suppoſe the Refraction of the Horizon

35m. cleared from Dip, then is the Sun's Centre in the Horizon when it appears 35m. above the Horizon. The like Method is to be obſerved for other Refractions near the Horizon; and therefore this Problem is uſeful for determining when an Obſervation ought to be made for the Amplitude.

Problem X. Having the Latitude of the Place and the Time elapſed from the Inſtant the Sun's Centre is in the true Horizon; to determine the Azimuth of the Sun.

This Problem hath two Caſes. 1ſt. When the Latitude and Declination are alike. 2d. When they are unlike. But previous to the Solution of this Problem, it will be neceſſary to know how to find the two Angles of an oblique ſpherical Triangle, when it's two Sides and the Angle contained between thoſe Sides are given.

In this Solution, the Sum of the two containing Sides muſt be leſs than 180 Degrees; if the Problem be ſuch that they are more, the ſupplemental Triangle to 180d. muſt be uſed in it's ſtead, and then of courſe, the ſupplemental Angle is found by the Proceſs. This makes no Uncertainty in the Solution, but requires a little Attention in the Computor, that the one be not taken for the other. The Theorem for the Operation is thus.

1ſt. As the *Sine* of the *half Sum* of the *containing Sides*, is to the *Sine* of *half their Difference*; ſo is the *Cotangent* of *half the contained Angle*, to the *Tangent* of *half the Difference* of the Angles ſought.

2d. As the *Coſine* of the *half Sum* of the *containing Sides*, is to the *Coſine* of *half their Difference*; ſo is the *Cotangent* of *half the contained Angle*, to the *Tangent* of *half the Sum* of the Angles ſought.

3d. The half Difference of the unknown Angles added to their half Sum, gives the greater Angle ſought, which is always oppoſite to the greateſt of the given ſides. And, the half Difference taken from the

half

half Sum, gives the leſſer Angle ſought, which is always oppoſite to the leſſer of the given Sides.

This Problem may be of great Uſe at Sea; for by the Methods before mentioned, the apparent Elevation which the Sun hath, when his Centre is in the Horizon, may be correctly aſſertained, and from that Inſtant till the Bearing of the Sun's Centre is obſerved by the Compaſs, the Time may be meaſured by the Ship's Clock or a Watch, and the Calculation may be eaſily made by the common logarithmic Tables, as in the following Example.

Caſe 1. Suppoſe the Latitude 50d. 0m. N. the Declination 12d. 18m. N. and the Time elapſed from the Sun's Centre being in the true Horizon to the Inſtant when his Bearing is obſerved 1h. 0m. of Time. Required the true Azimuth?

Solution. In the Triangle *bdc*, *dc* is 50d. 0m. *db* 77d. 42m. and the Angle at *c* 90d. Therefore, the Tangent of 50d. 0m. with the Co-tangent of 77d. 42m. makes the Coſine of the Angle *bdc* with Radius; hence the Angle *bdc* is 74d. 56m. which added to the Angle *bde* 15d. 0m. makes the Angle *cde* 89d. 56m. which taken from 180d. gives the Angle *edf* 90d. 4m. Then, having the Co-latitude *df*, the Polar diſtance *de*, and the included Angle *edf*, one of the other Angles *dfe* being found, is 81d. 59m. or *ac* the Azimuth required. In going through this Proceſs, and every Example of this kind, the Logarithms of the Sines and Coſines are eaſily taken out by the ſame Inſpection, alſo the Co-tangents; which greatly facilitates the Operation. Thus,

The half Sum of the Sides is 58d. 51m. and their half Difference 18d. 51m. The Sine of 58d. 51m. and it's Coſine are taken out at once. Alſo, the Sine of 18d. 51m. and it's Coſine at once. The remaining is very eaſy and conciſe. But what makes it preferable to other Methods, is its Accuracy; for by this Method, no Error can ariſe through horizontal Vapours, when the true Horizon hath been well aſcertained, and the Obſerver

Obſerver may meaſure out what Interval of Time he judges moſt proper.

Caſe 2. Is when the Latitude and Declination are unlike; that is, one North, the other South. In this Caſe, the ſupplemental Triangles to 180 Degrees are to be applied in the Solution, although they fall below the Horizon toward the depreſſed Pole; and when the Angle is found, the Figure will ſhew what is the true Angle, and what the ſupplemental one, to avoid any Uncertainty.

Problem XI. Having the ſame things given as in the former Problem, and the Sun's Bearing by the Compaſs at the ſame Time; to determine the Variation.

Rule. Firſt find the true Bearing or Azimuth, as in the former Problem; and compare it with the magnetic Bearing, ſo the Variation is had which was required.

Problem XII. Having two Altitudes of the Sun taken on the ſame Day, and the Interval of Time between thoſe Obſervations, meaſured by the Ship's Clock or by a Watch, to determine the true Azimuth or Bearing of the Sun, at each of the Times of Obſervation.

This Problem hath four Caſes, the manner of ſolving which is ſhewn at large in *Practical Navigation*, under the Articles *of Time at Sea*; where alſo it is ſhewn how to find the Solar Time at the Ship, and the Latitude of the Place, from the ſame Data; with the Corrections for the Horizon and other things too large to be diſcuſſed in a few Words, and therefore Reference is had to that Work for thoſe Particulars.

Problem XIII. Having the ſame things given as in the former Problem, and the Bearing of the Sun's Centre taken at each of theſe Obſervations, by the Compaſs; to determine the Variation.

Rule. Having found the true Azimuth by the former

mer Problem, compare it with the magnetic Azimuth, and it gives the Variation required.

This Problem will be uſeful at Sea, when the Latitude is quite unknown, or when the Ship hath been driven for a conſiderable Time, under ſuch Courſes and Diſtances, that no Dependance is to be had on the Latitude inferred from her Reckoning. It may alſo be uſeful on Land, where the Latitude is not known, nor the Interval a Perſon hath for being there, ſufficient for taking it by meridian Altitudes of the Sun or Stars.

Problem XIV. Having the Latitude of a Place, the Declination of a Fixed Star, and the elapſed Time from it's Riſing or Setting, till it is obſerved; to determine the true Azimuth at the Time of Obſervation.

This Problem is ſolved in the ſame manner as that for the Sun, having the ſame Data.

Problem XV. Having the ſame things as in the former Problem; alſo the Bearing of the Star by the Compaſs, to determine the Variation.

This Problem is alſo ſolved in the ſame manner as that for the Sun, having the ſame Data.

Problem XVI. Having a Variation Chart, to determine the principal Lines expreſſed on it, and how they are to be reckoned or counted.

A Variation Chart of the magnetic Needle, hath three kinds of Lines; namely, 1ſt. Of Latitude. 2d. Of Longitude. 3d. Of Variation. The Latitudes increaſe by Degrees and Minutes or Parts of a Degree from the Equinoctial to the Poles; theſe in Wright's Conſtruction are ſtraight Lines. The Longitudes increaſe both eaſtward and weſtward from that Meridian where Longitude begins, and therefore do not exceed 180 Degrees eaſtward or weſtward. The Variation Lines of equal Variation, croſs or run through the Chart, under ſuch Directions, that at any given Latitude and Longitude on the Chart, the Variation-line near that Point,

Point, ſhews the Variation of the Compaſs or Needle, at that Place of the Earth or Sea.

Problem XVII. Having the Latitude and Longitude of a Place, and a Variation Chart; to determine the Variation.

Rule. Guide your Eye either northward or ſouthward, amongſt the Parallels of Latitude, alſo eaſtward or weſtward amongſt the Meridians, until you come to the given Latitude and Longitude, and the Variation-lines near it will give the Anſwer required.

Problem XVIII. Having the Latitude and Variation, alſo a correct Variation Chart; to find the Longitude.

Rule. Guide your Eye northward or ſouthward, amongſt the Parallels of Latitude; alſo under ſuch other Direction as the Variation-lines require, until you come to the given Latitude and Variation, and amongſt the Meridians will be ſhewn the Longitude Eaſt or Weſt which was required.

Problem XIX. Having the Longitude and Variation, alſo a correct Variation Chart; to find the Latitude.

Rule. Guide your Eye eaſtward or weſtward amongſt the Meridians, alſo under ſuch other Direction as is ſuitable amongſt the Variation-lines, until you come to the Place where they meet; and by the Parallels of Latitude, the Latitude will be ſhewn which was required.

Problem XX. Having two equal Altitudes of a Star, the one taken a little before, and the other as much after the Meridian Tranſit; alſo, the Star's Declination and the elapſed Time between the two Obſervations; to determine the true Azimuth.

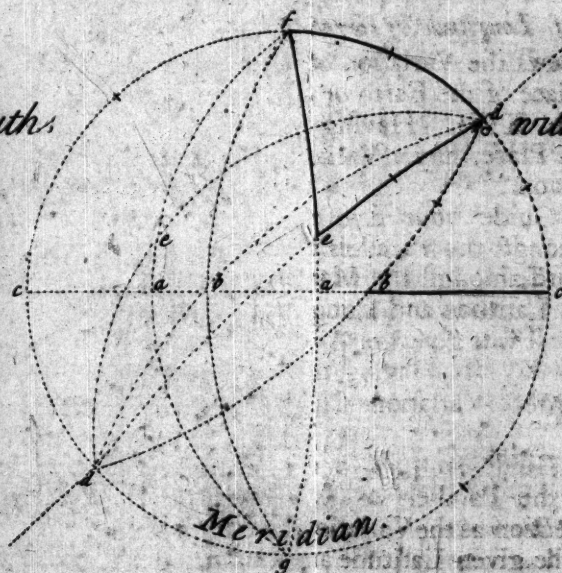
Rule. Find the Star's Polar-diſtance as is directed for the Sun; then ſay, as the Coſine of the Altitude is to the Sine of half the elapſed Time; ſo is the Sine of the Polar-diſtance, to the Sine of the Azimuth required.

Example. Suppoſe in an unknown Latitude the Altitude

The Azimuth without Altitude.

Given,
dc Latitude.
db a Pole,
de a Pole,
∠ bde.

Required,
the Azimuth
∠ dfe or ca
Also ae the
Altitude.



Calculation.

dc Tang.	50° 0′	10.07619	
db Co tan.	77. 42	9.33853	
∠ bde Cos	74. 56	9.41472	
∠ bde ddd	15. 0		
Sub	89. 56		
	180. 0		
∠ edf	2) 90. 4		
its half	45. 2		

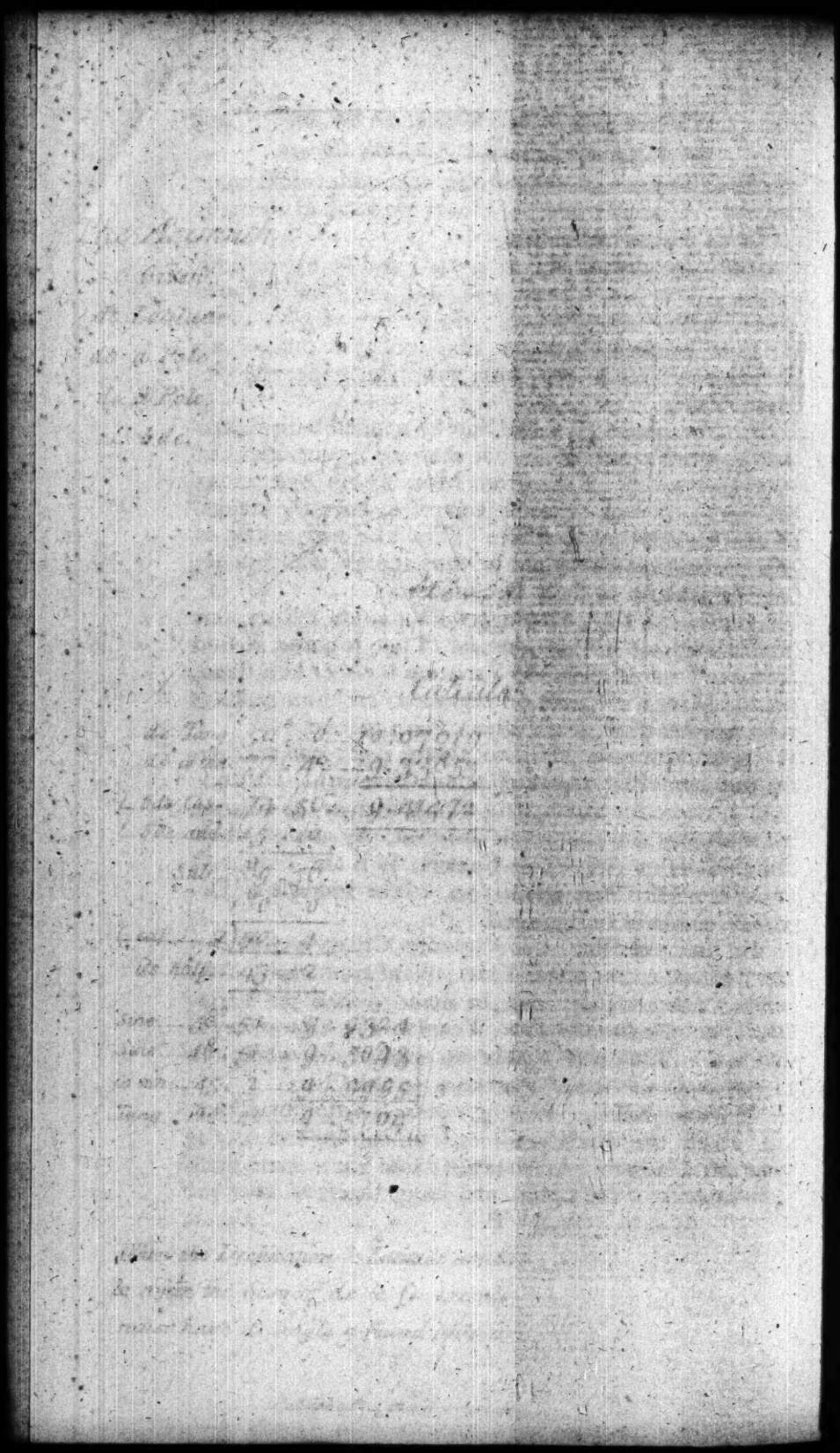
Sine	58° 51′	9.9324	
Sine	18. 51	9.5093	
Co tan	45. 2	9.9995	
Tang	20. 40	9.5764	

de	77° 42′	
df	40. 0	
2)	117. 42	
	58. 51	half Sum Sides.
2)	37. 42	
	18. 51	half diff. Sides.

Co sine	58° 51′	9	7137
Co sine	18. 51	9	9761
Co tan	45. 2	9	9995
Tang	61. 19	10	2619
	20. 40		
	81. 59	∠ efd required	

When the Declination & Latitude are unlike be will fall below the Equator, & when the Sum of de & fe exceeds 180, the supplemental Triangle deg must have its angle g found, this is supplement to dfe.

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titude of Syrius was observed 14 degrees before it came to the Meridian, and 5 Hours after it had the same Altitude past the Meridian.

Here the elapsed Time is 75d. and its half is 37d. 30m. The Co altitude 76d. and the Polar distance 106d. 25m. As Sine of 76d. to Sine of 37d. 30m. so Sine of 106d. 25m. to Sine of 37d. 0m. or it's Supplement 143d. 0m. from the Point under the elevated Pole.

In this manner may the Sun be applied with sufficient accuracy when he is not near the Equinoctial, be the Interval of Time from Morning till Afternoon; otherwise it will be most correct to have the elapsed Time as short as possible. The like may be said of the Moon, but she cannot be thus applied whilst under a great Change of daily Declination.

Problem XXI. Having two Variation Charts, one for Time past and another for Time to come, to find in what Parts of them the Variation is either at a stand, or in Motion eastward or westward, and how much it changes per Year at any given Place thereof.

Rule. Compare the two Charts with one another, at any two Places on each which have equal Latitude and Longitude; and having found what is the Change of Variation in the whole Interval, say as that whole Interval is to the whole Change, so is the Time past since the first Chart was drawn, to the Increase or Decrease of Variation required.

By this Method, two Variation Charts well drawn for 10, 20, 30, or more Years distant from each other, and to a large Scale, may be made to shew the Variation for the intermediate Years to any Exactness required. But there will be no need of this where the Lines continue almost stationary on the Chart.

Problem XXII. Having a correct Variation Chart, in which the Variation-lines terminate or end on the line of a Coast; to continue those Lines some small Distance over the Land, and judge thereby what the

Variation may be at Places near the Sea and at Places near the Rivers.

Rule. Obſerve the Flexure or Bend of the Variation-curves which are near the given Places, and continue them as nearly as poſſible in the Order which they appear to be continuing, ſo they will paſs over the given Places, and give the Variation required.

Problem XXIII. To apply a Variation Chart in the Practice of Navigation, and make it ſubſervient for determining the Longitude more correctly than can be otherwiſe expected by the uſual Method of Sailing.

Rule. Take out the Variation every Day or every half Day if required, for the Latitude and Longitude of the Ship as ſhewn by the Reckoning, allow this in the Courſe, together with Leeway and Currents, and the Longitude by the Reckoning will be as much corrected thereby, as if there could be a daily Obſervation of the Variation by an Amplitude or an Azimuth.

1ſt. This Problem is of ſuch extenſive Utility, that if there were no other Uſe of a Variation Chart, this alone would be ſufficient to make it of the greateſt Conſequence in Practical Navigation.

2d. The Time in which a Ship may ſail from the Lizard to the Iſland Madeira, under favourable Winds and other Circumſtances, is not more than ten Days; and could ſhe purſue her Voyage by the ſhorteſt Tract, in ſailing nearly five times that Diſtance, or in fifty Days, ſhe might arrive at the Cape of Good-hope, which is nearly the Mid-way to Bengal, this is near a fifth Part more than to the Malabar Coaſt, and about a fifth Part leſs than to China. But by reaſon of contrary Winds and other unfavourable Circumſtances, a longer Time is generally neceſſary for thoſe Voyages.

3d. In ſailing from the Lizard to the Cape, when the ſhorteſt way is not attempted, the Variation changes from 22 Degrees to 0 decreaſing, and again from

0 to

0 to 22 Degrees increaſing. Alſo, in ſailing from the Cape toward India, the Variation increaſeth from 22 Degrees to 28 Degrees; and then decreaſeth from 28 Degrees to 0.

4th. In unfavourable Weather it frequently happens, that for ten Days one after another, and ſometimes for a longer Time together, none of the Celeſtial Bodies are viſible, and therefore cannot be applied for obſerving either the Longitude or the Latitude; and therefore in ſuch Intervals, the Method of finding the Longitude by a Time-keeper cannot be practiſed, nor that by the Sun, Moon and Zodiacal Stars.

5th. In all ſuch Intervals of Time, the Ship may purſue her Voyage by the common Method of ſailing and the Uſe of a correct Variation Chart, as directed in this Problem, and not only correct the Longitude, but alſo the Latitude come to, by ſuch an Application of the Chart.

6th. This may be done for the longeſt Voyages, which will not only ſhorten them, but tend to preſerve Ships from running on Places of Danger, and return ſafely in the ſhorteſt Time and eaſieſt manner at the leaſt Expence.

Problem XXIV. To examine a Variation Chart of the Magnetic Needle, and determine whether the Lines of Variation thereon, be drawn truely or not.

In order to the Performance of this Problem, it is neceſſary to conſider how the Variation is obſerved at Sea; what Certainty there may be concerning the Latitudes and Longitudes of the Places, where Obſervations for the Variation have been made; and what Number of ſuch Obſervations is requiſite for proving the Truth or Errot of the Chart, with the Circumſtances attending them.

1ſt. Of the manner how the Variation of the Compaſs is commonly obſerved at Sea.

When an Amplitude is taken at Sea, the uſual Method is to obſerve when the Sun's Centre is in the viſible Horizon, by thoſe who make no Allowance for

Refraction; but when that Centre is about a Diameter of the Sun above the Horizon, when the Refraction is allowed for; both of which Methods are incorrect in an oblique Sphere, as is proved in *Practical Navigation*. For the ſame Reaſons, the Azimuth is frequently obſerved incorrectly, and the Error ariſing from theſe Cauſes alone, may frequently be more than that ariſing through the Motion of the Ship and the Card.

2d. Of the Latitudes and Longitudes of Places.

The former of theſe may often be known as correctly as is wanted, but the latter is often taken for the Truth when it is very erroneous, and this will be a Prejudice againſt the Variation. There are but three other ways, by which Longitude can be aſcertained, according to the uſual Practice, namely that by the Ship's Reckoning; by help of a Watch, and by the Lunar Method; the firſt of theſe is often erroneous; the ſecond may be ſo too, although the Watch be at mean Solar Time, both at the Beginning and End of the Voyage; and the Lunar Method may not give the Longitude more correctly than is conſiſtent with the difficulty of the Practice, and the ſeveral ſmall Errors incident to the various Parts of the Problem itſelf. If an Error by either of theſe Methods ſhould amount to 30 Miles in Longitude, no Variation Chart can be either examined or tried by it, but ſome other Method is wanting which may be depended on with more Certainty.

3d. Of the Number of Magnetic Variations obſerved.

In order to draw the former Variation Charts, "*near one hundred thouſand Obſervations*" were thought neceſſary, as cited page 59 *Practical Navigation*, conſequently, as great a Number for examining them, by the Reckonings, and as theſe happened to be imperfect, the Charts muſt neceſſarily have partaken in the Imperfection. By help of a Time-keeper, ſuch Obſervations would require many Years Application, and in the Interval, the Variation-curves would be altered, ſo that no Charts could be drawn by them; and a ſmall

Number

Of Longitude by correct Variation Charts.

Numbers of Obſervations would be inſufficient for drawing or examining the Curves. If the Lunar Method by Sun, Moon, and Zodiacal Stars be applied, the Difficulties will be nearly the ſame; and therefore all three of thoſe Methods put together, are not ſufficient for either examining or delineating correct Variation Charts of the Oceans, for Uſe in Navigation.

4th. All theſe Difficulties are eaſily overcome by the Diſcovery of a genuine Theory of the Variation, and the Laws by which thoſe Curves may (by help of a few Obſervations made at Places whoſe Latitudes and Longitudes have been well determined) be drawn and continued to great Diſtances on the Oceans; and by ſuch Laws, the *New Variation Charts*, have been drawn and publiſhed for the Uſe of Navigators.

Problem XXV. To ſhorten a long Voyage, or ſail from and return to a given Port, in the ſhorteſt Time and the ſafeſt manner poſſible.

This Problem is of ſuch extent, that in order to the performance of it in all its different Caſes, there is required a compleat knowledge of the Latitude and Longitude at Sea, and all the other Parts of Practical Navigation, with their Uſe and Application, when and where they happen to be wanted.

1ſt. It is particularly to be underſtood, what are the Advantages or Diſadvantages to be expected from Winds and other Incidents peculiar to the Oceans and Seas which are to be croſſed, in order to make uſe of one or avoid the other.

2d. What are the Lands, Iſlands, Rocks, Shoals, Sands or other Obſtructions in the Line of the Ship's Tract, leading from one Place to another, and what are the beſt Methods of avoiding them.

3d. What are the Courſes to be ſailed from one place to another, where the Ship muſt paſs, if ſhe purſues her Voyage by the ſhorteſt Diſtances in the leaſt Time; and how her Navigators are not to be deceived in

in any part of such Tracts, either in the Latitude or Longitude.

4th. What Advantages may be derived from keeping near the Arch of a Great Circle, if the Voyage extends toward the Polar Regions, and what Difference there is between the Arch of a Great Circle and the Rhumb Line, in sailing near the Equinoctial, or in the Temperate Zones.

5th. Under due consideration of these and some other particulars of lesser consequence, the Navigator will direct the Ship by the nearest Distances that can be passed with safty, and in the Practice will frequently do well to keep as much to one and the same constant Course, as the Winds and other Circumstances will permit.

6th. But whatever the Course be on which the Ship sails, in the midst of the Oceans, it will be absolutely necessary to know the Variation of the Compass every Day, in many Cases it will be wanted at the end of every 12 Hours, otherwise the Course intended to be sailed will become incorrect, and the Ship will pass by places which were not intended to be in her Tract; and as the Variation cannot every Day be taken or observed, it can no other way be known but from a correct Variation Chart drawn for that Time and Place.

7th. Whilst the Ship is thus on a long Voyage, by contrary Winds, she will frequently be driven from the Tract intended to be kept; in such Cases, either of the usual Methods of finding the Longitude is to be applied and the best Methods of determining the Latitude, by which the Errors introduced into the Reckoning, will be discovered, and the nearest or best Course again assumed for arriving at the intended Port.

8th. From these Considerations, it appears that whatever Courses may be most desirable for the Performance of long Voyages, the Practice of Navigation will not permit the Ship to keep to them; the whole Ocean

Of Longitude by correct Variation Charts.

Ocean itſelf is that ſpacious Field over which a Ship is frequently under the neceſſity of making large Traverſes, although at the Time ſhe leaves the Land, it is intended that ſhe ſhall only croſs the Ocean; and therefore, the Navigator ſhould be equally acquainted with all Incidents throughout this Field, and be equally prepared againſt it's Accidents.

Problem XXVI. Having two Altitudes of the Sun taken on the ſame Day, and the Interval of Time between the Obſervations meaſured by the Ship's Clock or a Watch, to determine the Time from Noon, the Azimuth and Latitude, ſuppoſing the Sun's Declination to have been nearly the ſame in that Interval.

In Article 81. page 69.; Art. 90. page 77. and Art. 95. page 110. Reference is had for the Solution of this Problem and other things depending thereon, to *Practical Navigation*; but as that Treatiſe may not be in the Hands of every Perſon, it ſeems proper to give a general account of the Problem and it's Solution, here.

When or by whom it was firſt invented is not certainly known, poſſibly it hath been of an earlier Date than is generally imagined, ſince we find it was propoſed and ſolved by Mr. John Seller Hydrographer to the King, in his Practical Navigation, Problem 23 of the Doctrine of the Sphere, about 105 Years ago. Since that Time it hath been revived, often publiſhed by Aſtronomical Writers, and recommended as of great Uſe in the Practice of Navigation; notwithſtanding which, to the preſent Time it hath not gained Credit enough with Navigators in general, ſo as to be brought into daily Uſe, but the old Method of taking the Latitude by the Sun's Meridian Altitude, hath been commonly conſidered as more certain.

About 20 Years ſince (as appears by a Pamphlet accompanying the Work) this Problem, after introducing additional Data, was propoſed and ſolved in a different manner, by Mr. Cornelius Douwes of Amſterdam; who according to the Preface of the Nautical Ephe-

meris

meris 1771 was rewarded for it, and his Tables when improved were publiſhed with the Ephemeris that Year. Since that, the firſt publiſhed Tables of Mr. Douwes have appeared in other Tracts, under the Pretence of making this Subject better ſuited for the Public, but thoſe in the Ephemeris are beſt.

As the Problem was firſt propoſed, by the two Altitudes and the elapſed Time, to determine the Time of each Obſervation, the Azimuth at each Obſervation, and the Latitude of the Place, it ſeemed to be fairly limited, and wanting nothing elſe to be added in order to get the ſeveral Anſwers required. But with theſe Data only, the Solution could not be had without going through the various parts of the ſpherical Triangles that were formed thereby in the Sphere. Alſo, before thoſe Triangles could be ſolved, it was neceſſary to correct the Effect of the ſpheroidal Horizon, and get the true Altitudes as they would have been obſerved on a ſpherical Earth, which being done, the Solution of the Triangles in the four different Caſes, will always be as follows.

Caſe 1. When the Latitude and Declination are alike, that is both North or both South, *P* will repreſent the elevated Pole, *Z* the Zenith, *ſ* the Sun at one Obſervation and *S* at another when the Altitudes have been taken both before or both after Noon on the ſame Day; conſequently *ZP* being a Part of the Meridian, *a ſ* is one Altitude and *ſ Z* it's Co-altitude; *b S* the other Altitude, and *S Z* its Co-altitude, and *ſP* or *S P* the Polar-diſtance. Let *S ſ* be joined by the Arth of a Great Circle, and thereon drop *P d* perpendicular, ſo will the Angles made thereby at *d* be right Angles, and in the two Triangles *SPd* and *ſPd* (each right angled at *d*) the Hypothenuſe or Polar-diſtance is given, and the Angle *SPd* or *ſPd* which is half of the elapſed Time. Hence the Arch of the Great Circle *Sſ* being found, by it and the two Co-altitudes *SZ* and *ſZ* an Oblique Triangle *S Z ſ* is formed. But in the right angled Triangle, the Angle *P ſ d* being found, and in the Obli-

que

que Triangle, the Angle $Z f S$, the Angle f being found, the one ſubtracted from the other leaves the Angle $P f Z$, with which and the two Sides $P f$ and $Z f$ a calculation being made gives the Angle $f P Z$, the Angle $P Z f$, and the Side PZ, that is the Hour from Noon and the Azimuth when the Sun was at S, and the Co-latitude of the Place of Obſervation.

Caſe 2. This is when any two Sides of the Oblique Triangle formed above the Horizon, together are more than 180d. Thus, if SP and ZP together make more than 180d. the ſupplemental Triangle $p S n$ is to be ſolved in it's ſtead, and then it will be found that the Angle at the depreſſed Pole will be the ſame as that at the elevated one, and the Angle at the Nadir equal to that at the Zenith; and the Angle S in both Triangles alike. But due Care ſhould be had to obſerve how thoſe Triangles are formed in the Sphere.

Caſe 3. The two firſt Caſes ſuppoſe the Obſervations to have been made whilſt the Sun was on the ſame Side of the Meridian. This third Caſe ſuppoſeth the Latitude and Declination alike but one Obſervation before and the other after Noon on the ſame Day. Here the Points S and f are ſuppoſedly joined by the Arch of the Great Circle as in the former Caſes, and the Angles of the right angled and oblique Triangles found as before; whoſe Difference gives the included Angle, either at f or S, from which and the two containing Sides the Hour Angle from Noon, the Azimuth and Co-latitude are found as before.

Caſe 4. This happens when one Obſervation is made before, the other after Noon on the ſame Day, and the Sum of any two Sides of a Triangle above the Horizon, are more than 180d. but the Sum of the ſupplemental Sides of the ſame Triangle below the Horizon are leſs than 180d. In this Caſe, the ſupplemental Triangle is to be computed and the Reſult ſtrictly attended to, according to the Formation of thoſe Triangles in the Sphere.

This is the manner of ſolving the Problem accord-

ing to its own original Data only, but under the additional Data, it takes another Form, and is thus,

Problem XXVII. Having the ſame things as in the former Problem, alſo the Latitude of the Place ſomewhat near the Truth to determine the true Latitude, or that agreeing with correct Obſervations.

This is the Problem on which ſo much has been written, and which is eſteemed of ſuch Uſe in Navigation. In it four things are given, the two Altitudes, the elapſed Time, and the Latitude ſomewhat near the Truth, to find the true Latitude. Three of theſe are all that are wanted, ſuppoſing a ſpherical Earth; the fourth or ſuppoſed Latitude happens here to be ſuch, that it helps to bring out a Solution more conciſely than by the original Triangles.

But however excellent this Method may be, it has it's own peculiar Qualities. 1ſt, It is only intended as a Method of Approximation. 2d. That Approximation preſuppoſeth the Latitude aſſumed within certain Limits of the Truth. 3d. One, two or more Computations muſt be made in all Approximations, where the firſt Operation is not ſufficient for determining the Truth. 4th. It requires it's own peculiar Tables, and without theſe nothing can be done; and the Conſtruction of theſe may be conſidered as myſterious to ſome good Mathematicians, much more to thoſe who want to make general Uſe of them. 5th. They require the Uſe of Natural Sines, which are not always at hand. 6th. A Tranſition from Natural to Artificial Numbers in the midſt of the Calculation, a Circumſtance which makes Computations inelegant. 7th. They are not general but limited to their own peculiar Times for the Obſervations. 8th. The Notes, Cautions and Remarks which have been publiſhed for rendering them familiar and eaſy, are not a few, nor eaſily to be underſtood and remembered by Mathematicians themſelves. 9th. They carry in their Proceſs by way of complication, the little Errors ariſing through the ſpheroidal Horizon, which cannot be diſſevered from them,

Of Longitude by correct Variation Charts.

them, however nearly they may agree with Obſervations. Theſe are ſome of the Circumſtances attending the Solution after the moſt approved Methods that have been publiſhed, whilſt the general one from it's own natural Principles hath long been neglected and uncultivated. But however great theſe Difficulties may have appeared they are eaſily overcome by the following Method, which may be illuſtrated by an

Example. Suppoſe the two Altitudes 46d. 20m. and 54d. 7m. each being taken either in the Forenoon or After-noon of the ſame Day, and the Sun's Declination 11d. 17m. N. the elapſed Time between the two Obſervations 1h. 45m. or 21d. 15m. and the ſuppoſed Latitude 46d. 20m. N.

Solution. Project the ſeveral Parts as per Scheme; then are *GO*, *YP*, *LQ*, *ZO*, *ip*, *lq* each repreſenting 90d. In the Right-angled Triangle *rzs*, *rz* being 46d. 20m. the Angle *zrp* is 12d. 4m.; but the Angle *NEP* is 0d. 41m. more, which being added to the former gives the Angle *NEP* 12d. 45m.; hence having *SN* and *SO* the Angle at *O* is found, and the Latitude 46d. 30m. which was required.

Whoever conſiders the Eaſe with which the Problem may be ſolved in this manner, will be aſtoniſhed at the Pains and Labour which our moſt reputable Mathematicians and Aſtronomers have taken to commenting on Approximation. This Method has been known to me a long Time, and it would not now have been printed, but to ſhew, that whilſt ſome Methods have met with all deſireable Encouragement, others have not been unknown; and when this Method comes to be conſidered more at large, what Difference there is between them, will be plainly ſeen by the Public.

Problem XXVIII. Having two Altitudes of a Fixed Star whoſe Declination is known, and the Sidereal Time elapſed between the two Obſervations; to find the Hour Angles, the Azimuth Angles, and the Latitude.

This Problem is like the former, only here the ſide-

rial Time is to be used, because each Hour of this Time makes 15 Degrees of the Equator. But when the Sun is used, each Hour of Solar Time makes 15 Degrees of the Equator very nearly. Hence it follows, that knowing how much a Planet or any other Phœnomenon gains or loses on either solar or sidereal Time in 24 Hours (whilst it's Declination continues the same and it is unaffected by Parallax) it may be applied in the same manner as the Sun or a Fixed Star.

Problem. XXIX. Having the Distance of a Planet from a Fixed Star near the Ecliptic, with the Planet's horizontal Parallax at the Time of Observation; also, the Altitude of the Planet and the Altitude of the Star at the same Time, to find the true Distance of their Centres.

In the Solution of this Problem, the horizontal Parallax of the Planet may sometimes be imperceptible at other Times not so, according to it's greater or lesser Distance from the Earth. As the Moon's Distance from the Earth is less than that of the Primary Planets, her horizontal Parallax will thereby be greater, but when the Primary Planets approach toward the Earth their horizontal Parallaxes must be increased accordingly, and hence will arise their Parallaxes in Altitude, by the same Proportion as is used in finding the Moon's Parallax in Altitude.

Having the Planet's Parallax in Altitude, the same Process is to be gone through, for finding the true Distance of Centres, as is used for those of the Sun and Moon; and if this be done at the end of short Intervals of Time, the Planet's true Place in the Ecliptic, may hereby be inferred for immediate Use, independant of Predictions by any astronomical Tables.

Problem. XXX. To apply the Planets instead of Fixed Stars, together with the Lunar Predictions, or any other astronomical Tables.

This may be done by correctly determining the Planets Places as directed in the former Problem, and then observing them and computing from the Observations

varions as though they were Fixed Stars. But it will be moſt exact to uſe thoſe which alter their Places ſloweſt, or which apparently continue whilſt they are thus uſed near the ſame Place in the Heavens.

Problem XXXI. To determine in what Oceans or Seas or any of their Parts, the Longitude may be found, and to what Exactneſs it may be taken in thoſe Places, by the Variation of the Compaſs and the obſerved Latitude.

This Problem requires the beſt Variation Charts that can be drawn for all Oceans and Seas frequented by Navigators, but without ſuch it cannot be determined whether the Longitude can be found in thoſe Places or not. If the Lines of Variation are ſuch on the Chart, that there is an Alteration of a Degree of Variation in 4 Degrees of Longitude, the Longitude may frequently be taken in ſuch Places within a Degree or 60 Miles, becauſe Experience proves, that in many Caſes, the Variation (being well obſerved at Sea) may not be erroneous more than a quarter of a Degree. But by the Theory of the Variation and alſo by Obſervations, it is certain, that there are many Places in the Oceans, where the Variation-lines are near four Times as favourable, and therefore at thoſe Places it be ſuppoſed the Longitude can be taken within half a Degree or 30 Miles, when a ſufficient allowance is made for the Errors commonly ariſing in the Obſervation and Computation.

XXVIII. *Of finding the Longitude at Sea univerſally.* By the four foregoing Methods, the Longitude at Sea hath been found, and again may be found whenever they can be practiſed; it is therefore well worth the Attention of a Navigator to ſtudy and practiſe each of them according to the beſt of his Power, and as the Circumſtances attending it will enable him; for although ſome Methods leading to this Practice may be attended with Expence others are very moderate, and no Expence can make a Method perfect which is erroneous in its Principles, but Induſtry and

Ingenuity

Ingenuity may in a great meaſure ſupply Defects where the Method requires but a few Inſtruments eaſily to be applied and eaſy Computations.

97th. If the Longitude were ſought by having the Moon's Altitude above the Horizon of the Sea, with other Data ſuppoſed to be ſufficient for determining either her Right aſcenſion or her Longitude, the apparent ſpheroidal Horizon muſt firſt be reduced to the true ſpherical one, beſides which the exact quantity of horizontal Refraction muſt be known at all times and Places where the ſhip happens to be. If 35 Minutes of a Degree be allowed for the Sun or Moon's Centre being above the true Horizon, when that Centre coincides with that Horizon, and at the ſame Time 27 Minutes is the true Allowance, the Difference is 8 Minutes, and the Luminaries ſo obſerved near the ſame vertical Circle, and computed for, would produce an Error of 4 Degrees of Longitude or 240 Miles at the Equinoctial. For this Reaſon, ſuch a Method of obſerving cannot but be attended with great Uncertainty and Error, and thoſe who firſt propoſed it cannot prove it ſo good as that by taking the Diſtance of the Sun and Moon or Moon and Star.

98th. If the Method by Jupiter's Satellites be wiſhed for at Sea, it will have many Imperfections attending it which are by ſome Perſons thought not to exiſt. For, 1ſt. Whatever Error is in the predicted Times of the Satellites appearing or diſappearing, will be an Error in the Longitude. 2d. The Teleſcope muſt be applicable at Sea. 3d. The Obſerver muſt be undeceived in all States of the Air, which of the Satellites is to be obſerved, and not miſtake one for the other, a Circumſtance attended with more Difficulty than is generally imagined. 4th. There are frequently no ſmall Errors happening by this Method, even when the Longitude is found by it on Land, for which ſee *Practical Aſtronomy*. 5th. The Attention of the Obſerver, muſt be kept up in as ſtrict a manner as in any kind of Obſervation whatſoever, on Land, and more

more would be wanted at Sea on Account of the Motion of the Ship. 6th. The Planet Jupiter is not always viſible in the Night on Account of his Proximity to the Sun. And other Reaſons might be given why this Method cannot become general if at all practical at Sea, however uſeful it may be on Land. And as to the Conjunctions of the Satellites or their apparent paſſing near one another, ſuch Obſervations may be made on Land and profitably applied, but the Motion of the Ship and a previous Knowledge of the Poſitions of the Satellites, will always introduce more Difficulties than can be well overcome at Sea, were there no other.

99th. It remains therefore to find the Longitude by one or other of the Methods before mentioned, and to improve each of them as much as poſſible. If the Lunar Method be farther improved when the predicted Places of the Sun and Moon are ſufficiently correct for the Longitude, the next additional Improvement would be, to take the Diſtance of the Sun and Moon or Moon and Star more correctly than can be done by Hadley's Sextant. Something of this kind was given out by me in Phil. Trans. 15 Years ſince, which was 5 Years before the Nautical Almanac began, but the Improvement was dropt for want of Encouragement.

As the Lunar Method is capable of Improvement ſo are the other three, and being improved would become of greater Uſe to the Public. One of the greateſt Improvements that has ever been added to the firſt Method, is the Invention of a Theory whereby to draw the Variation-lines over the Surface of the Oceans; for the Errors ariſing from Leeway and Currents may often be diſcovered by help of the obſerved Latitude; and whoever ſuppoſes the Variation may be taken ſo as to ſupply the Uſe of a Variation Chart, ſuppoſes nothing but fair ſunſhine during a Voyage, which is contrary to Experience. Alſo, the Variation or fourth Method admits of greater Improvement than has been hitherto even wiſhed for; and it is not difficult to form a

Judgment

Judgment what may be nearly the ſtate of the ſecond Method, under the greateſt Encouragement it can receive from the Public.

100th. But to form a Judgment concerning the Merit of each of the four Methods, it is proper to examine, how frequently and to what Accuracy each of them may be practiſed when and where they are wanted at Sea, becauſe from theſe Circumſtances their Utility will ariſe, and it will appear how much Navigation is aſſiſted by each of them.

Advantages of the firſt Method.

1ſt. This Method the Navigator muſt be acquainted with, whatever other he thinks proper to practiſe at particular times and places. 2d. It is eaſy and for ſhort Voyages the moſt certain of all the Methods, when practiſed with Care and the lateſt Improvements. 3d. It is univerſal, may be applied at all times as well by Night as by Day, and a Voyage is never hindred in it for want of ſeeing the Sun. 4th. It is the only Method whereby the direct Courſe of a Ship can be known, to ſail the ſhorteſt way toward the intended Port. 5th. In it, by help of the obſerved Latitude, the ill Effect of Currents and Leeway may often Times be diſcovered, and thereby the Longitude nearly determined. 6th. It is the only Method whereby the Variation, Currents, Leeway, Latitudes and Longitudes, and other Incidents can be examined throughout a ſmall Part of an Ocean or Sea, when a ſtationary Place thereof is well aſcertained.

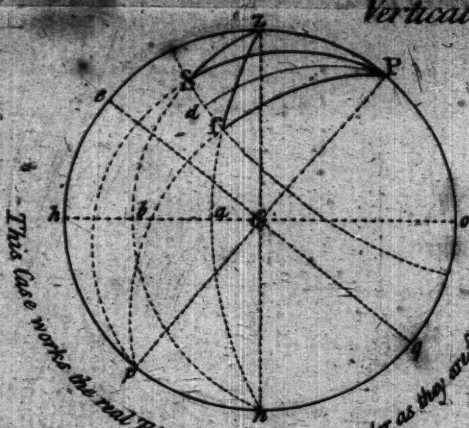
Diſadvantages of the firſt Method.

1ſt. It is liable to Error at the End of a long Voyage; and ſometimes in a ſhort Interval great Errors will unavoidably creep into the Reckoning. 2d. Whatever Error ariſeth in this Method, and cannot be corrected from it's own Principles, the Ship carries it along with the Reckoning and one of the other Methods muſt be uſed for correcting ſuch Errors, otherwiſe the Ship may be in Danger or prolong her Voyage.

Advan-

Case 1.

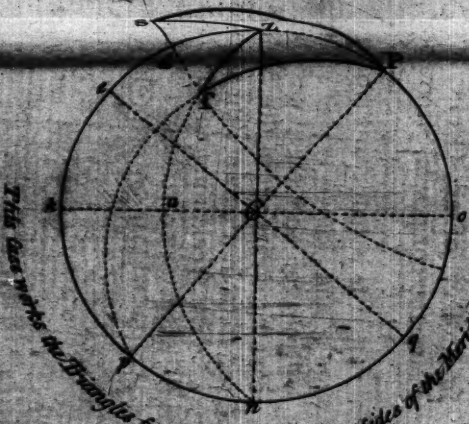
Vertical and Polar Spherica
applicable at Sea



This case works the real Triangles in the same Order as they are formed per Figure.

These four Cases, for taking the Times, Azimuths & Latitude are fully explained in my Treatise of Practical Navigation.

Case 3.



This case works the Triangles formed, but on different Sides of the Meridian of the Place.

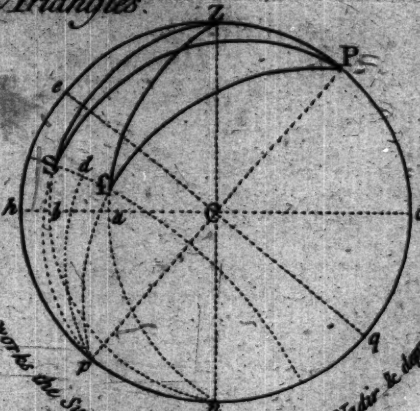
N.B.
P denotes the elevated Pole, whether it be North or South.

J. Dunn delin.t

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oherical Triangles.
at Sea.

Case 2. Plate 13

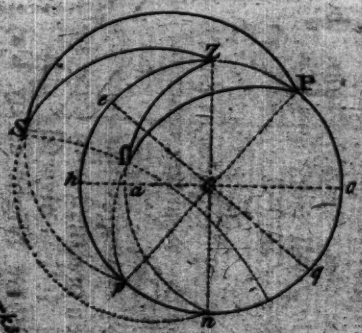


This Case works the Supplemental Triangles, to the Natur & depressed Pole.

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latitude;
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Case 4.

S L is supposedly joined by the Arch. of a Great Circle on which the Perp.r from P falls.



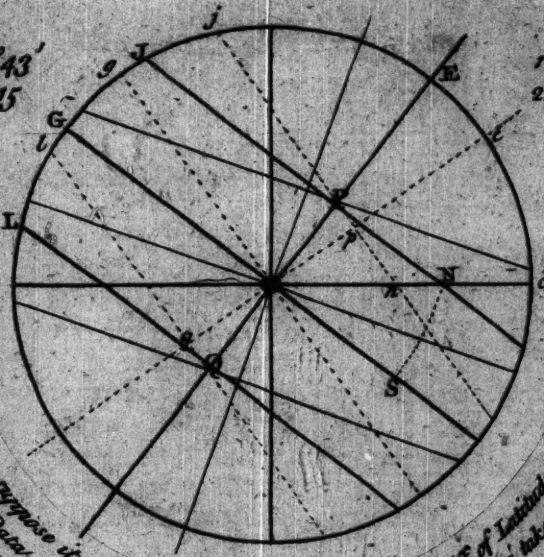
This Case works the Supp.al Triangles on different Sides of the Meridian of the Place.

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Pole,
t be
outh.

ment Nov.r 17.th 1777. by S. Dunn.

Data.

Polar dist. 78°. 43′
Elap. time 21. 15



Assume the Latitude twice, as suppose it is 46°. 20′ or 46°. 36′, with which & the Data, get the true hour Angles, & the false elapsed times will be 20°. 34′ & 21°. 42′, the diff. is 68′ & the diff. of Latitude 16′. From the true elapsed time 21°. 15′ take 20°. 34′ remains 41′ which added to 15′, the Latitude of the Place assumed gives the true Latitude 46°. 36′ nearly.

Data.

1st Co alt. 43°. 5′
2d Co alt. 35. 53

Calculation.

Lat.	46 . 20		Lat.	46 . 20	
Co. lat.	43 . 40	Co. ar. 0 . 16086	Co. lat.	43 . 40	Co. ar. 0 . 16086
à Pole	78 . 43	Co. ar. 0 . 00848	à Pole	78 . 43	Co. ar 0 . 00848
Co. alt	43 . 5		Co. alt.	35 . 53	
2	165 . 28		2	158 . 16	
	82 . 44	Sine 9 . 99650		79 . 8	Sine 9 . 99214
	39 . 39	Sine 9 . 80489		43 . 15	Sine 9 . 83581
	2	19 . 97073		2	19 . 99729
	14 . 47	9 . 98536		4 . 30	9 . 99864
L	29 . 34	is 20°. 24′ Diff.	L	9 . 0	

Lat.	46 . 36		Lat.	46 . 36	
Co. lat.	43 . 24	Co. ar. 0 . 16299	Co. lat.	43 . 24	Co. ar 0 . 16299
à Pole	78 . 43	Co. ar. 0 . 00848	à Pole	78 . 43	Co. ar 0 . 00848
Co. alt	43 . 5		Co. alt	35 . 53	
2	165 . 12		2	158 . 0	
	82 . 36	Sine 9 . 99637		79 . 0	Sine 9 . 99195
	39 . 31	Sine 9 . 80366		43 . 7	Sine 9 . 83473
	2	19 . 97150		2	19 . 97815
	14 . 36	9 . 98575		3 . 45	9 . 98907
L	29 . 12	is 21°. 42′ Diff.	L	7 . 30	



Of finding the Longitude at Sea univerſally.

Advantages of the ſecond Method.

1ſt. This Method is very eaſily practiſed. 2d. It requires only a Sight of the Sun when the Latitude is known, and otherwiſe two Sights of the Sun and the elapſed Time, will determine both the Latitude and Longitude.

Diſadvantages of the ſecond Method.

1ſt. The Time-keeper muſt be ſuch a curious piece of workmanſhip, ſo fit for the purpoſe, and requiring ſo much ingenuity, ſkill and care, that the Artiſt who makes it, will be juſtly intitled to ſuch a Price for it as the Work deſerves. 2d. That Price may be unſuitable with many Perſons, who want to find the Longitude at Sea. 3d. Others who do not regard the Expence, may not want the Longitude either by Sea or Land; ſo that but a few of the beſt of thoſe Machines can be applied in the Sea-ſervice. 4th. The Machine muſt have a ſtrict Trial before it is taken to Sea, and that can no way be effected with Certainty but by the moſt perfect aſtronomical Obſervations. 5th. If it happens to err at Sea, nothing belonging to it can ſhew what the Error is, or point out a Correction. 6th. It may go well for a conſiderable Time, yet alter by Cauſes or Accidents unforeſeen, and ſometimes with Difficulty accounted for.

Advantages of the third Method.

1ſt. This Method, in it's moſt ſimple Form, requires no better Inſtrument than what a Ship ought to have for finding the Latitude, if it be deſigned for a long Voyage. 2d. Nor better Ability than Navigators have in general, for making Obſervations. 3d. The Longitude may be taken by this Method, both in the Day and in the Night. 4th. In the Night Obſervation, the Longitude may be taken, frequently, by help of a Star eaſt, and alſo one weſt of the Moon; and the Medium of the two may happen to correct an Error in either. 4th. In making the three cotemporary Obſervations, either by Day or Night, there is no Difficulty in taking the Altitude of either the Sun, Moon or Star,

to ſuch Exactneſs as is wanted. 5th. This Method is general, when the proper Luminaries appear, and therefore may be of great Uſe after a long Voyage, when Ships happen to be near Land. 6th. Large Ships are ſeldom unprovided with more than one Obſerver with a good Inſtrument; and two or three Obſervations at the ſame Time, may give a Medium more exact than a ſingle one by itſelf. 7th. Perſons who are a little acquainted with the Principles, will eaſily judge when the Obſervations may be ſometimes ſo made, as to need but little (if any) Correction for Refraction and Parallax. 8th. And at other Times, ſcarce any thing elſe but for Refraction and Parallax in Altitude; which reduces the Problem to more ſimple Terms than it hath hitherto been ſuppoſed capable of.

Diſadvantages of the third Method.

1ſt. The Obſervation may not be made when either of the Luminaries is near the Horizon, without ſuppoſing the obſerved Diſtance of Centres uncertain, on account of the horizontal Vapours. 2d. Near the Horizon, the Refraction may be different at different times and places. 3d. During 2, 3, or 4 Days in a Month, the Moon may be too near the Sun to admit of Obſervation. 4th. For 8 or 9 Days in a Month, the Sun and Moon will be too far diſtant for an Obſervation in the Day-time, and then it will be moſt eaſy to carry Solar Time till Night by help of a Watch, and obſerve by the Moon and Zodiacal Stars. 5th. For about 12 Days in a Month the Day-obſervation cannot be made for the foregoing Reaſons, and during the other 18 Days, there may be either favourable or unfavourable Weather. 6th. When the Ship is not near the Equinoctial, Care muſt be taken not to ſettle the Time at the Ship by an Altitude of the Sun near the Meridian. 7th. The ſeveral ſmall Errors, ariſing in the Conſtruction and Uſe of the Sextant, or from any other Cauſes, whenever they together on one Side amount to a Minute of a Degree, will introduce an Error of half a Degree in Longitude. 8th. There is no probable Method of expunge-

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Of finding the Longitude at Sea universally.

ing such errors; because in a Series of Events, the same never returns but at the end of certain Intervals of Time; so that if one happens to be right, there is a Probability that the next may be erroneous.

Advantages of the fourth Method.

1st. This Method requires no other Instruments, Books nor Instructions, than what Navigators are or ought to be acquainted with in the Practice of the first Method, nor no uncommon Ability to make Use of them. 2d. When the Latitude is nearly known, an Observation for the Longitude, may be made by this Method, at any Time when ever the Sun is visible, provided he is not very near the Meridian. This is peculiar to no other Method but that of a Time-keeper, by help of a single Observation of the Sun. 3d. The Observation may frequently be made by a Sight of such a Fixed Star as is well situated for the Purpose, and many of these may be applied. 4th. This Method is not dependant on a previous knowledge of the Longitude of any other Place, beside that where the Ship is at the Observation. 5th. The Circumstances of the Theory are numerous, and set aside Objections from a few imperfect Observations. 6th. In the midst of the Oceans, for vast Tracts or Spaces, the Longitude may be taken by this Method within a Degree, sometimes perhaps within half a Degree or less; and this likewise, at some Places near the Lands. 7th. This Method is attended with less Expence and Care than any other Method. 8th. The first Method is subject to the greatest Errors without the Assistance of the Charts used in this Method; and with their Assistance, it is probable that first Method may be nearly perfected.

Disadvantages of the fourth Method.

1st. The Lines of Variation, do, at some Places, run such Lengths near the Parallels of Latitude; that the Longitude cannot be found, by help of the Variation and observed Latitude. Or what is attended with equal Disadvantage; they may be widely distant from

each other, although nearly parallel with the Meridians. 2d. Theſe Inconveniences may happen at Places in the Oceans, where Ships want to paſs; but the whole of them together, bear a ſmall Proportion to the whole of the other Places, where Ships paſs in the moſt important Voyages.

101ſt. By comparing the Advantages and Diſadvantages in each of the four Methods with each other reſpectively, it will not be difficult to form a Judgment concerning the Utility of each of them; but as they happen to be of different kinds, Experience is that alone from which a juſt Concluſion can be drawn.

However uſeful and perfect the firſt Method is in ſhort Voyages, it is liable to Error of no little Conſideration in long Voyages; and therefore, in ſuch, it is deemed an inſufficient or precarious Method, which is intended to be ſupplied by the ſecond, third, or fourth Method; in which Senſe, theſe three, or any other Method, can be conſidered only as ſupplemental to the firſt method. Nevertheleſs, there are Advantages on the whole, peculiar to each of the Methods, and they will appear by a Compariſon of them.

Whilſt the ſecond Method is general and eaſy, what will it be when the Time-keeper happens to faulter, or otherwiſe to be in the hands of a Perſon that is not over-nice in determining the Rate of its daily gaining or loſing? this Care is not wanting in either the third or fourth Method.

In the third Method (though the Ephemeris be not wanting, nor ſkill and ability to uſe it and he Sextant) for two fifths of the Time the Ship is at Sea, and probably near half the Voyage, the Obſervation in the Day-time may not be made on account of the Moon's neareſt Approach to and greateſt Receſſion from the Sun; and although all that Time, there may be Obſervations in the Night, a good Watch to carry Time will be of great Uſe to all who are not compleatly ſkilled in the Sphere, and if that Machine ſhould fail, however perfect the Method is, thoſe Circumſtances

cumſtances may take off a part of it's Utility. For, the want of Opportunity to practiſe a Method, is equally diſadvantageous as being unprovided with one.

In the fourth Method, the Ship may want to ſail through an Archipelago, in which the Variation-lines run unfavourable for the Longitude; and which (were that Method only truſted to) might be attended with more diſagreeable Circumſtances than in the midſt of an Ocean. But in ſuch Places, the knowledge of the Variation will always be a great Addition to the firſt Method, and by that Method only, can all minute Particulars be tranſacted. If a tenth or any other part of all the Oceans, were attended with ſuch Diſadvantages, or rather ſuch a part of the Oceans and Seas as is croſſed in a long Voyage (letting aſide particular Circumſtances) as great Imperfection may ariſe in either of the other Methods, for want of being able to obſerve during the ſame proportional part of the Voyage; but it may be farther noted, that in the wide Oceans, ſuch places can be avoided, whilſt there is no Method of obſerving the Celeſtial Bodies, when they diſappear through unfavourable Weather.

102d. Here might be introduced a great Number of Proofs concerning the Truth of the firſt Variation-lines drawn almoſt wholly from the Theory, by comparing them with the moſt accurate Obſervations otherwiſe made at Sea; but it is judged proper to omit a full account thereof in this Work, and to give but a few Inſtances only, as follows.

1771. April 28th. At Sea, in Lat. 27d. 6m. S. and 36d. 12m. W. from Point Palmiras; the Beginning of the Lunar Eclipſe was obſerved at 16h. 28m and the Variation obſerved 21d. 0m. W.

The Beginning of this Eclipſe reduced to St. Paul's London by the Connoiſſance des Temps was at 13h. 3m. 30s. which taken from 16h. 28m. gives 3h. 24m. 30s. or 51d. 7m. the Longitude from London.

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The Longitude of Point Palmiras is 87d. 15m. E. from London, from which take 36d. 12m. gives 51d. 3m. the Longitude by the Ship's Reckoning.

The Variation 21d. W. and Lat. 27d. 6m. S. gives Longitude 50d. 12m. which is but 47 nautical Miles more Weſt than the Medium by the Prediction of the Eclipſe and the Ship's Reckoning.

The foregoing Obſervation was communicated to me, by a young Gentleman whoſe Name I have not leave to mention, and ſince my Charts were drawn; but I do not ſuppoſe any extraordinary care was taken in the Obſervation, though it anſwers ſo well; other Inſtances might be produced, to prove, that in a great part of this vaſt Ocean, the Longitude is not only diſcoverable within a Degree, but within half a Degree by the Variation and Latitude only.

103d. If the Obſervations made by other Methods throughout the Atlantic, Ethiopic and Indian Oceans, do not exactly agree with the Charts; although the Differences happen to be on one Side, it is not always a Proof that the one is right or the other erroneous. The firſt Compariſons I made in this manner ſoon after the Charts were drawn, proved this Truth; for (as hath been before explained) there is no certain Method whereby they can be cenſured, whilſt the Errors by them are not greater than may reaſonably be expected in the other Methods, by which they are examined. In a great Number of Inſtances, the Charts prove no way inferior to any other Method in point of Exactneſs. Theſe few Inſtances are from Obſervations by Gentlemen whoſe Names it ſeems improper for me here to mention.

Latitudes. D. M.	Var. obſerved. D. M.	Long. by Moon D. M.	Diff. of Moon & Chart.
34. 55S.	4. 0W.	14. 8W.	6 Miles, minus.
11. 4S.	4. 50W.	25. 18W.	3 Miles, minus.
31. 28S.	8. 52W.	8. 33W.	2 Miles, minus.
32. 37S.	12. 51W.	0. 29W.	1 Mile, plus.
34. 4S.	19. 10W.	13. 54E.	1 Mile, plus.

33. 9S.

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33. 9S. — 21. 30W. — 18. 23E. — 6 Miles, plus.
34. 0S. — 22. 0W. — 18. 42E. — 0
3. 39S. — 12. 45W. — 48. 52E. — 6 Miles, plus.
2. 28S. — 11. 13W. — 50. 32E. — 6 Miles, plus.
38. 45S. — 25. 12W. — 28. 50E. — 4 Miles, plus.

If theſe Inſtances are not enough, many more may be produced, from real Obſervations made in the moſt exact manner with the beſt Inſtruments. But notwithſtanding the near agreement between theſe or any others, one or the other of two Obſervations differing conſiderably from each other may be nearer the Truth, and which is the right, it may be very difficult to determine.

XXIX. *Of Tables for the Longitude at Sea.*

104th. There are ſeveral different kinds of Tables uſed in finding the Longitude at Sea, each of which belongs to it's own peculiar Method, or rather they are employed in ſolving the Triangles and computing other things which are required according to the Method practiſed. In the firſt Method, the Tables commonly uſed in Navigation are applied for determining the Difference of Latitude and Difference of Longitude; beſide which, the Logarithms are neceſſary for computing the Amplitude and Azimuth, and the Tables of Declination for ſhewing the Latitude by help of Meridian Altitudes, or otherwiſe by two Altitudes and the elapſed Time.

The ſecond Method requires the Logarithms and the Declinations, to make Computation for the Time at the Ship. And, the fourth Method requires the ſame Tables, for computing the true Amplitude and Azimuth at the Time of Obſervation.

But, the third Method requires. 1ſt. The Ephemeris for ſhewing the ſeveral Data computed from the Aſtronomical Tables. 2d. The Logarithms for computing the Time at the Ship, and for other things. 3d. The Requiſite Tables for ſhortning the Calculation of the Time at the Place for which the Ephemeris

was

was made. The latter may be either of the voluminous or contracted kind. But obſerving the Inſtructions in this Treatiſe, their Uſe may be wholly ſupplied (when the Refraction is known) by the common logarithmic Tables only.

105th. Therefore the Variation Method of finding the Longitude at Sea, requires no more Tables than the Watch Method; and wherever the Variation Chart ſhews the Longitude as truly as the Watch, it is there as aſſiſting. But neither the firſt, ſecond, nor fourth Methods, require the logarithmic Tables nicer than to each Minute of a Degree; and if the third Method be applied after the manner directed in this Work, all the Computations of that Method may be made by the ſame Tables. To which may be added; that,

In the original Method of Calculation, theſe Caſes of Plain and Spherical Triangles will ſolve a multitude of uſeful Problems.

1ſt. In a right-angled Plane Triangle, either of it's Sides may be Radius, and the other Sides will be Sines, Tangents or Secants; and what Proportion the Side made Radius hath to Radius; the ſame have the other Sides to the Sines, Tangents or Secants by them repreſented. 2d. In an oblique angled Plane Triangle; as the Baſe is to the Sum of the Sides; ſo is the Difference of the Sides to the Difference of the Segments of the Baſe. 3d. In a right angled Spherical Triangle (turning the three parts fartheſt from the right Angle into Complements) the Sine of a Middle Part with Radius; makes the Tangents of the Extremes conjunct; and the Coſines of the Extremes disjunct. 4th. In the oblique Spherical Triangles, as the Tangent of the half Baſe, to the Tangent of the half Sum of the other Sides; ſo is the Tangent of the half Difference of thoſe Sides, to the Tangent of the half Difference of the Segments of the Baſe. Theſe, and other Theorems, are at large in *Practical Aſtronomy*.

End of the Precepts.

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End of the Contents of the Precepts.

☞ Here follow the Tables, uſed together with the common Logarithms and the Ephemeris, by perſons who do not chooſe to compute from original Principles according to the Inſtructions in this work.

Theſe Tables will be uſeful in computing the Longitude by help of the Sun, Moon and Zodiacal Stars; thus, 1ſt. By the Table of the Sun's Parallax in Altitude, the few ſeconds on that account are taken out at Sight. 2d. By the Table of the Moon's Parallax in Altitude for the given Minutes of horizontal Parallax, the Parallax in Altitude is taken out at ſight, and for any greater horizontal Parallax a proportional Part is found and added thereto. 3d. The Increaſe of the Moon's Semidiameter for her Altitude is taken out at Sight. 4th. How the Refraction near the Horizon is determined hath been already ſhewn; which, if it agree with the Tables, the effect is to be taken out at Sight, otherwiſe ſuch Method is to be obſerved as directed by the Tables. The Uſe of the other Tables is either obvious or explained farther on.

I. A TABLE

I. A TABLE, shewing the Sun's Parallax in Altitude, suppoſing the horizontal Parallax either 8, 8½, or 9 Seconds of a Degree.

8 Seconds.			8½ Seconds.			9 Seconds.		
D.	M.	S.	D.	M.	S.	M.	D.	S.
Alt. 0.	0	8	Alt. 19.45		8	Alt. 27.15		8
29.	0	7	34.35		7	38.55		7
41.25		6	45.	5	6	48.10		6
51.20		5	54.	0	5	56.15		5
60.	0	4	61.55		4	63.35		4
68.	0	3	69.20		3	70.50		3
75.30		2	76.25		2	77.10		2
82.50		1	83.15		1	83.35		1
90.	0	0	90.	0	0	80.	0	0

Parallax in Altitude in Seconds

In this Table, the firſt Column of Degrees and Minutes expreſſeth the Altitude of the Sun's Centre, as ſuppoſed unaffected by Refraction; oppoſite to which, in the ſecond Column is the Parallax in Altitude to each Second, from the Horizon to the Zenith. This Table is computed by Article 52, alſo by Problem 25. page 83. for the neareſt 5 Minutes of a Degree.

III. A TABLE ſhewing how many Seconds of a Degree are to be added to the Moon's horizontal Semidiameter, to have her true Semidiameter, at every 5 Degrees from the Horizon to the Zenith; for the leaſt, mean and greateſt horizontal Semidiameters or horizontal Parallaxes.

D.	S.	D.	S.	D.	D.
Alt. 0	0	Alt. 0	0	Alt. 0	0
5	1	5	1	5	2
10	2	10	2	10	3
15	4	15	4	15	4
20	5	20	6	20	6
25	6	25	7	25	8
30	7	30	8	30	9
35	8	35	9	35	11
40	9	40	10	40	12
45	10	45	11	45	13
50	11	50	12	50	14
55	11	55	13	55	15
60	12	60	14	60	16
65	13	65	15	65	17
70	13	70	16	70	18
75	13	75	16	75	18
80	14	80	16	80	18
85	14	85	16	85	19
90	14	90	17	90	19

Seconds to be added to the horizontal Semidiameter.

In this Table the firſt column expreſſeth the Moon's Altitude, oppoſite to which in the Second Column, are the Seconds to be added to the Moon's horizontal Semidiameter, and it gives the true Semidiameter of the Moon at that Altitude.

II. A TABLE of the Moon's Parallax in Altitude, to each Degree, for the horizontal Parallax 54 Minutes.

	D.	M. S.	Seconds.	Diff.
Alt.	0	54. 0	3240	
	1	53.59	3239	1
	2	53.57	3237	2
	3	53.55	3235	2
	4	53.52	3232	3
	5	53.47	3227	5
	6	54.42	3222	5
	7	53.36	3216	6
	8	53.28	3208	7
	9	53.20	3200	8
	10	53.10	3190	10
	11	53. 0	3180	10
	12	52.49	3169	11
	13	52.37	3157	12
	14	52.23	3143	14
	15	52. 9	3129	14
	16	51.54	3114	15
	17	51.39	3099	15
	18	51.22	3082	17
	19	51. 4	3064	18
	20	50.45	3045	19
	21	50.25	3025	20
	22	50. 4	3004	21
	23	49 42	2982	22
	24	49.20	2960	24
	25	48.56	2936	24
	26	48.32	2912	24
	27	48. 7	2887	25
	28	47.41	2861	26
	29	47.14	2834	27
	30	46.46	2806	28

In this Table, the firſt Column expreſſeth the Degrees of Altitude and the ſecond the Parallax in Altitude ſuppoſing the horizontal Parallax 54 Minutes; theſe are reduced to Seconds in the third Column and in the fourth are the Differences. When the Logarithms are not at hand, the Parallax in Altitude may be found for any horizontal Parallax, thus. For Example, ſuppoſe the horizontal Parallax 48m. 30s. and the Altitude 14d. 30m. and the Parallax in Altitude is required? Here, by Inſpection the Parallax in Altitude is 3136 Seconds to the horizontal Parallax 2240 Seconds; but the given horizontal Parallax is 3510 Seconds. Therefore, as 3240 to 3136 ſo 3510 to 3398 Seconds or 56m. 38s. the Parallax in Altitude required. The ſame may be found by other Methods and this Table, but this ſeems the plaineſt; nevertheleſs, the Logarithms are certainly beſt when applied as directed in Article 52.

Radius	90d. 0m.	10.000
to Coſine	14d. 30m.	9.9859
ſo hor. par.	3510m.	3.5453
to par alt.	3398m.	3.5312

Whoever conſiders the Eaſe and Correctneſs of this Method, will prefer it to any other for finding the Parallax in Altitude; for it eaſily gives it to the neareſt Second of a Degree.

The true Dip of Horizon, or Allowance for the Height of the Eye above the Surface of the Sea, to be ſubtracted from the obſerved Altitude is thus.

1	foot, 1 min.	32	feet, 6 min.
3¼	feet, 2 min.	43	feet, 7 min.
8	feet, 3 min.	57	feet, 8 min.
14¼	feet, 4 min.	72	feet, 9 min.
22	feet, 5 min.	89	feet, 10 min.

Note. In uſing the following Tables of Refraction, if the Refraction at the Horizon be leſs than that in this Table, find it in the ſecond Column; and as many Minutes as are oppoſite thereto in the firſt Column, add to the given Altitude and for the Total take out the Refraction required.

II. A TABLE of the Moon's Parallax in Altitude, to each Degree, for the horizontal Parallax 54 Minutes.

Alt.	Par. in Alt. D.	M.	S.	Seconds.	Diff.
	30	46.	46	2806	
	31	46.	17	2777	29
	32	45.	47	2747	30
	33	45.	17	2717	30
	34	44.	46	2686	31
	35	44.	14	2654	32
	36	43.	41	2621	33
	37	43.	7	2587	34
	38	42.	33	2553	35
	39	41.	58	2518	35
	40	41.	22	2482	36
	41	40.	45	2445	37
	42	40.	8	2408	37
	43	29.	29	2369	39
	44	38.	50	2330	29
	45	38.	10	2290	40
	46	37.	29	2249	41
	47	36.	49	2209	40
	48	36.	8	2168	41
	49	35.	25	2126	42
	50	34.	42	2083	43
	51	33.	59	2039	44
	52	33.	14	1995	44
	53	32.	30	1950	45
	54	31.	44	1904	46
	55	30.	58	1858	46
	56	30.	11	1811	47
	57	29.	24	1764	47
	58	28.	37	1716	48
	59	27.	48	1668	48
	60	26.	59	1619	49

Alt.	Par. in Alt. D.	M.	S.	Seconds.	Diff.
	60	26.	59	1619	
	61	26.	10	1570	49
	62	25.	21	1521	49
	63	24.	31	1471	50
	64	23.	40	1420	51
	65	22.	49	1369	51
	66	21.	57	1317	52
	67	21.	3	1265	52
	68	20.	13	1213	52
	69	19.	21	1161	52
	70	18.	28	1108	53
	71	17.	35	1055	53
	72	16.	41	1001	54
	73	15.	48	948	53
	74	14.	54	894	54
	75	14.	0	840	54
	76	13.	6	786	54
	77	12.	11	731	55
	78	11.	16	676	55
	79	10.	21	621	55
	80	9.	25	565	56
	81	8.	29	509	56
	82	7.	33	453	56
	83	6.	27	387	56
	84	5.	41	341	56
	85	4.	44	285	56
	86	3.	47	228	57
	87	2.	50	171	57
	88	1.	53	114	57
	89		56	57	57
	90		0	0	57

IV. A Table of Refraction in Altitudes.

Refraction.			Refraction.			Refraction.			Refraction.		
D.	M.	M. S.	D.	M.	M. S.	D.	M.	M. S.	D.	M.	M. S.
0.	0	34.27	0.	30	28.57	1.	0	24.32	1.	30	21.16
	1	34.15		31	28.47		1	24.24		31	21.10
	2	34. 3		32	28.37		2	24.17		32	21. 4
	3	33.51		33	28.28		3	24.10		33	20.58
	4	33.39		34	28.18		4	24. 3		34	20.52
	5	33.27		35	28. 9		5	23.56		35	20.46
	6	33.15		36	27.59		6	23.49		36	20.40
	7	33. 3		37	27.49		7	23.42		37	20.34
	8	32.51		38	27.40		8	23.35		38	20.28
	9	32.40		39	27.31		9	23.28		39	20.22
	10	32.29		40	27.23		10	23.22		40	20.16
	11	32.17		41	27.13		11	23.15		41	20.10
	12	32. 6		42	27. 4		12	23. 8		42	20. 4
	13	31.55		43	26.55		13	23. 1		43	19.58
	14	31.44		44	26.46		14	22.55		44	19.52
	15	31.33		45	26.37		15	22.49		45	19.46
	16	31.22		46	26.28		16	22.42		46	19.41
	17	31.11		47	26.19		17	22.35		47	19.36
	18	31. 0		48	26.10		18	22.29		48	19.31
	19	30.49		49	26. 2		19	22.23		49	19.26
	20	30.39		50	25.54		20	22.17		50	19.21
	21	30.29		51	25.45		21	22.10		51	19.16
	22	30.18		52	25.36		22	22. 4		52	19.11
	23	30. 8		53	25.27		23	21.58		53	19. 6
	24	29.57		54	25.18		24	21.52		54	19. 1
	25	29.47		55	25.10		25	21.46		55	18.56
	26	29.37		56	25. 2		26	21.40		56	18.51
	27	29.27		57	24.54		27	21.34		57	18.46
	28	29.17		58	24.46		28	21.28		58	18.41
	29	29. 7		59	24.39		29	21.22		59	18.36
	30	28.57	1.	0	24.32		30	21.16	2.	0	18.31

IV. A Table of Refraction in Altitude.

Refraction.		Refraction.		Refraction.		Refraction.	
D. M.	M. S.	D. M.	M. S.	D. M.	M. S.	D. M.	M. S.
2. 0	18.31	2. 30	16.13	3. 0	14.28	3. 30	12.50
1	18.27	31	16. 8	1	14.24	31	12.47
2	18.23	32	16. 3	2	14.20	32	12.44
3	18.19	33	15.59	3	14.16	33	12.41
4	18.15	34	15.55	4	14.12	34	12.38
5	18.11	35	15.51	5	14. 8	35	12.35
6	18. 6	36	15.47	6	14. 4	36	12.32
7	18. 1	37	15.43	7	14. 0	37	12.29
8	17.56	38	15.39	8	13.56	38	12.26
9	17.51	39	15.35	9	13.53	39	12.23
10	17.46	40	15.31	10	13.50	40	12.20
11	17.41	41	15.37	11	13.47	41	12.18
12	17.36	42	15.33	12	13.44	42	12.16
13	19.31	43	15.29	13	13.41	43	12.14
14	17.26	44	15.25	14	13.38	44	12.12
15	17.21	45	15.21	15	13.35	45	12.10
16	17.16	46	15.17	16	13.32	46	12.08
17	17.11	47	15.13	17	13.29	47	12.16
18	17. 6	48	15. 9	18	13.26	48	12.14
19	17. 1	49	15. 5	19	13.23	49	12.12
20	16.57	50	15. 1	20	13.20	50	12.10
21	16.53	51	14.57	21	13.17	51	11.58
22	16.49	52	14.53	22	13.14	52	11.56
23	16.44	53	14.50	23	13.11	53	11.54
24	16.39	54	14.47	24	13. 8	54	11.52
25	16.34	55	14.44	25	13. 5	55	11.50
26	16.30	56	14.40	26	13. 2	56	11.48
27	16.25	57	14.37	27	12.59	57	11.46
28	16.21	58	14.34	28	12.56	58	11.44
29	16.17	59	14.31	29	12.53	59	11.42
30	16.13	3. 0	14.28	30	12.50	4. 0	11.40

IV. A TABLE of Refraction in Altitude.

Refraction.		Refraction.		Refraction.		Refraction.	
D. M.	M. S.	D. M.	M. S.	D. M.	M. S.	D. S.	M. S.
4. 0	11.40	4. 30	10.40	5. 0	9.40	5. 30	8.51
1	11.38	31	10.38	1	9.38	31	8.50
2	11.36	32	10.36	2	9.36	32	8.49
3	11.34	33	10.34	3	9.34	33	8.48
4	11.32	34	10.32	4	9.32	34	8.47
5	11.30	35	10.30	5	9.30	35	8.46
6	11.28	36	10.28	6	9.28	36	8.45
7	11.26	37	10.26	7	9.26	37	8.44
8	11.24	38	10.24	8	9.24	38	8.43
9	11.22	39	10.22	9	9.22	39	8.42
10	11.20	40	10.20	10	9.20	40	8.41
11	11.18	41	10.18	11	9.18	41	8.40
12	11.16	42	10.16	12	9.16	42	8.39
13	11.14	43	10.14	13	9.14	43	8.38
14	11.12	44	10.12	14	9.12	44	8.37
15	11.10	45	10.10	15	9.10	45	8.36
16	11. 8	46	10. 8	16	9. 8	46	8.35
17	11. 6	47	10. 6	17	9. 6	47	8.34
18	11. 4	48	10. 4	18	9. 4	48	8.33
19	11. 2	49	10. 2	19	9. 2	49	8.32
20	11. 0	50	10. 0	20	9. 0	50	8.31
21	10.58	51	9.58	21	8.59	51	8.30
22	10.56	52	9.56	22	8.58	52	8.29
23	10.54	53	9.54	23	8.57	53	8.28
24	10.52	54	9.52	24	8.56	54	8.27
25	10.50	55	9.50	25	8.55	55	8.26
26	10.48	56	9.48	26	8.54	56	8.25
27	10.46	57	9.46	27	8.53	57	8.24
28	10.44	58	9.44	28	8.52	58	8.23
29	10.42	59	9.42	29	8.51	59	8.22
30	10.40	5. 0	9.40	30	8.51	6. 0	8.21

IV. A TABLE of Refraction in Altitude.

D.	M.	Refraction. M. S.	D.	M.	Refraction. M. S.	D.	M.	Refraction. M. S.	D.	M.	Refraction. M. S.
6.	0	8.21	6.	30	7.46	7.	0	7.15	7.	30	6.45
	1	8.19		31	7.44		1	7.14		31	6.44
	2	8.17		32	7.43		2	7.13		32	7.43
	3	8.15		33	7.42		3	7.12		33	6.42
	4	8.13		34	7.41		4	7.11		34	6.41
	5	8.11		35	7.40		5	7.10		35	6.40
	6	8.10		36	7.39		6	7. 9		36	6.39
	7	8. 9		37	7.38		7	7. 8		37	6.38
	8	8. 8		38	7.37		8	7. 7		38	6.37
	9	8. 7		39	7.36		9	7. 6		39	6.36
	10	8. 6		40	7.35		10	7. 5		40	6.35
	11	8. 5		41	7.34		11	7. 4		41	6.34
	12	8. 4		42	7.33		12	7. 3		42	6.33
	13	8. 3		43	7.32		13	7. 2		43	6.32
	14	8. 2		44	7.31		14	7. 1		44	6.31
	15	8. 1		45	7.30		15	7. 0		45	6.30
	16	8. 0		46	7.29		16	6.59		46	6.29
	17	7.59		47	7.28		17	6.58		47	6.28
	18	7.58		48	7.27		18	6.57		48	6.27
	19	7.57		49	7.26		19	6.56		49	6.26
	20	7.56		50	7.25		20	6.55		50	6.25
	21	7.56		51	7.24		21	6.54		51	6.24
	22	7.54		52	7.23		22	6.53		52	6.24
	23	7.53		53	7.22		23	6.52		53	6.24
	24	7.52		54	7.21		24	6.51		54	6.24
	25	7.51		55	7.20		25	6.50		55	6.24
	26	7.50		56	7.19		26	6.49		56	6.24
	27	7.49		57	7.18		27	6.48		57	6.24
	28	7.48		58	7.17		28	6.47		58	6.24
	29	7.47		59	7.16		29	6.46		59	6.24
	30	7.46	7.	0	7.15		30	6.45	8.	0	6.24

IV. A TABLE of Refraction in Altitude.

Refraction. D.	M.	S.	Diff.	Refraction. D.	M.	S.	Diff.	Refraction. D.	M.	S.	Diff.
8	6	24	8.	35	1	17	8.	62	0	29	
9	5	43	41	36	1	14	3	63		28	1
10	5	10	33	37	1	12	2	64		27	1
11	4	43	27	38	1	9	3	65		26	1
12	4	19	24	39	1	7	2	66		25	1
13	3	58	21	40	1	5	2	67		23	2
14	3	40	18	41	1	2	3	68		22	1
15	3	24	16	42	1	0	2	69		21	1
16	3	9	15	43		58	2	70		20	1
17	2	56	13	44		56	2	71		19	1
18	2	45	11	45		54	2	72		18	1
19	2	35	10	46		52	2	73		17	1
20	2	26	9	47		51	1	74		16	1
21	2	18	8	48		49	2	75		15	1
22	2	11	7	49		47	2	76		14	1
23	2	5	6	50		46	1	77		13	1
24	2	0	5	51		44	2	78		12	1
25	1	55	5	52		42	2	79		11	1
26	1	50	5	53		41	1	80		10	1
27	1	46	4	54		39	2	81		9	1
28	1	41	5	55		38	1	82		8	1
29	1	37	4	56		37	1	83		7	1
30	1	33	4	57		35	2	84		6	1
31	1	29	4	58		34	1	85		5	1
32	1	26	3	59		33	1	86		4	1
33	1	23	3	60		31	2	87		3	1
34	1	20	3	61		30	1	88		2	1
35	1	17	3	62		29	1	89		1	1

Note. The Seconds of Refraction are taken out by Inspection when the Altitude is under 8 Degrees; but when above they are found by a Part proportional.

FINIS.